Optimal Dynamic Carbon Taxes in a Climate-Economy Model with Distortionary Fiscal Policy

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Abstract

How should carbon be taxed as a part of fiscal policy? The literature on optimal carbon taxes generally abstracts from other taxes and prescribes *Pigouvian* carbon levies. However, when governments raise revenues with distortionary taxes, carbon levies have fiscal costs and benefits. While they raise revenues directly, they may simultaneously shrink the bases of other taxes (e.g., by decreasing employment). This paper theoretically characterizes and then quantifies optimal carbon tax schedules in a climate-economy model with distortionary fiscal policy. The macroeconomic setup is a dynamic general equilibrium model with linear taxation. The environmental setup is a state-of-the-art representation of the carbon cycle and climate-economy feedbacks based on the DICE framework. First, this paper establishes a novel theoretical relationship between the optimal taxation of carbon and of capital income. This link arises because carbon is conceptually equivalent to negative capital: Emissions accumulate in the atmosphere and decrease output. Quantitatively, the welfare costs of *not* taxing carbon and of taxing capital income are large and of similar magnitude ($25 trillion, $2005 lump-sum consumption-equivalent). Second, this study demonstrates that optimal carbon taxes must internalize climate change production impacts (e.g., on agriculture) and direct utility impacts (e.g., on biodiversity existence value) differently. Third, this paper compares the setting with distortionary taxes to the setting with lump-sum taxes considered in the literature. The central quantitative finding is that optimal carbon tax schedules are $20-35\%$ lower when there are distortionary taxes. This adjustment produces a welfare gain of $600 billion to $1.5 trillion, depending on the structure of income taxes.

1 Introduction

Raising revenues and addressing climate change are two fundamental challenges facing governments. This paper considers these tasks jointly. Specifically, I study the optimal design of carbon taxes both as an instrument to control climate change and as a part of fiscal policy.

The Intergovernmental Panel on Climate Change (IPCC) recently concluded that "warming of the climate system is unequivocal," based on detailed reviews of the scientific literature (IPCC WG I, 2007). Moreover, "most" of the observed temperature increase is "very likely" due to anthropogenic greenhouse gas emissions, most importantly carbon dioxide (IPCC WG I, 2007). Climate change is expected to affect human welfare through numerous channels. These include changes in agricultural productivity, sea-level rise, ocean acidification, species extinctions, increased extreme weather events, disease vector changes, and others (IPCC WG II, 2007). The prices of carbon-based fuels do not currently reflect these external costs in all but a few countries.[1]

Both academic[2] and policy[3] studies of optimal carbon pricing generally focus on this market failure as the only distortion in the economy. In such a setting, the optimal carbon tax is *Pigouvian*. This levy internalizes the environmental damage costs of carbon emissions.[4] However, theses studies do not consider potential interactions between carbon levies and other taxes.

Carbon levies, if implemented, will interact with existing tax policy (e.g., taxes on labor, capital income, and consumption). On the one hand, carbon taxes raise revenues directly. On the other hand, they may decrease revenues indirectly by shrinking the bases of other taxes. For example, if climate policy decreases employment, this will reduce the revenue benefits and exacerbate the welfare costs of labor taxes. Several studies have argued that the welfare costs of these fiscal interactions likely exceed the (non-environmental) revenue benefits of carbon taxes (Goulder, 1995; Bovenberg and Goulder, 1996; Jorgenson and Wilcoxen, 1996; Babiker, Metcalf, and Reilley, 2003). Bovenberg and Goulder (1996) consequently advocate taxing carbon below Pigouvian rates. However, these papers do not incorporate the environmental benefits of climate policy. That is, they do not consider feedback effects between the climate and the economy.

This paper theoretically characterizes and then quantifies optimal carbon tax schedules in an integrated assessment climate-economy model (IAM) with distortionary fiscal policy. I combine a dynamic general equilibrium model of the world economy that includes linear taxes with a state-of-the-art representation of the carbon cycle and climate-economy feedbacks based on the seminal DICE framework (Nordhaus, 2008). All IAMs necessarily represent the world in simplified terms.

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[1] Countries with carbon taxes include Sweden, Finland, and Denmark (see Sumner, Bird, and Smith, 2009).
[2] Nordhaus (2008), Golosov, Hassler, Krusell, and Tsyvinski (2011), Acemoglu, Aghion, Bursztyn, and Hemous (2011), Hope (2006), Manne and Richels (2004), Tol (1997), etc.
[3] E.g., U.S. Interagency Working Group (2010).
[4] Specifically, the Pigouvian tax equals the social cost of carbon - the value of marginal damages from another ton of carbon emissions - evaluated at the optimal allocation.

However, IAMs are the best available tool to value the impacts of carbon dioxide emissions. These effects may last over extremely long time horizons (Archer, 2005). Their valuation thus requires macroeconomic models that predict economic activity over long time scales to the best of our ability and current knowledge. The three main findings of this paper are as follows.

First, I establish a novel theoretical relationship between the optimal taxation of carbon and of capital income. Intuitively, the climate is an asset used in production (e.g., of agriculture). Carbon emissions accumulate in the atmosphere and change the climate, with adverse effects on output. Giving up consumption to reduce emissions thus yields a future return of avoided production damages. In other words, the climate is an *environmental capital* good.[5] I show that setting carbon taxes below Pigouvian rates distorts incentives to invest in climate capital. This is analogous to capital income taxes, which distort incentives to invest in physical capital.[6] The first main result is as follows: If it is optimal to set capital income taxes to zero, then the optimal carbon tax fully internalizes production damages at the Pigouvian rate, even if labor markets are distorted. This is because both policies reflect the government's desire to leave investment undistorted. The literature on optimal dynamic Ramsey taxation has demonstrated the desirability of undistorted savings decisions in a wide range of models (Chamley, 1986; Judd, 1985; Atkeson, Chari, and Kehoe, 1999, Acemoglu, Golosov, Tsyvinski, 2011, etc.). My result shows that the logic against capital income taxes extends to environmental capital.

Second, I demonstrate that optimal carbon taxes must value climate damages that affect production differently from those that affect utility directly. Utility impacts reflect the value of the climate as a final consumption good (e.g., biodiversity existence value). Internalizing these damages yields no production gain and creates efficiency costs due to tax interactions. Consequently, I show that the optimal carbon tax does not fully internalize utility damages, taxing them below the Pigouvian rate. This result formally extends Bovenberg and Goulder's (1996) classic optimal tax formulation to a dynamic setting with capital and carbon accumulation. Their formulation and much of the literature[7] on pollution control in the presence of distortionary taxes assumes that environmental degradation affects only utility. However, I further find that the benchmark formulation does not extend to output damages.[8] This finding arises because of the climate's role as an input to production. A central result of optimal commodity taxation theory states that intermediate input usage should not be distorted, because such distortions reduce production efficiency (Diamond and Mirrlees, 1971). In line with this theorem, I find

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[5]   The notion of the climate as environmental capital is standard in the literature (see, e.g., Nordhaus, 2010).
[6]   Specifically, both policies result in a wedge between the intertemporal marginal rates of substitution and transformation.
[7]   E.g., Parry, Williams, and Goulder, 1999; Bovenberg and Goulder, 1996; Goulder, 1995; Bovenberg and de Mooij, 1994; Sandmo, 1975, etc.
[8]   As discussed below, in a static setting, this result was previously demonstrated by Williams (2002) and Bovenberg and van der Ploeg (1994).

that output losses from climate change are fully internalized through a Pigouvian tax in the benchmark setting.[9,10]

Third, I use my model to compare optimal climate policy in the setting with distortionary taxes to the setting with lump-sum taxes considered in the literature. I find that the optimal carbon tax schedule is $20 - 35\%$ lower when there are distortionary taxes. Two effects of distortionary taxes explain this result. One, they decrease the size of the economy and hence the value of climate damages. Two, they alter the optimal carbon tax formulation to charge less than the full value of marginal damages. Optimal carbon prices start at $43 - 55 per metric ton of carbon ($2005/mtC$) in 2015 and rise to $426 - 541/mtC$ by 2105. The upper end of this range reflects a full tax reform scenario, in which the government simultaneously optimizes over capital, labor, and carbon taxes. The lower end reflects a green tax reform scenario, in which capital and labor income taxes continue at suboptimal business-as-usual levels, but optimized carbon levies are added to the tax code.[11]

These three results further relate to the literature in the following ways.

First, the carbon-capital tax link is a novel result, to the best of my knowledge. On the theory side, a rich literature has explored pollution taxes in a setting with distortionary taxes (reviewed by Bovenberg and Goulder, 2002). However, this literature has predominantly focused on static models. As a result, few studies in this area have considered intertemporal distortions and their effects on dynamic processes, such as carbon or capital accumulation.[12] Several studies have modeled pollution levies in endogenous growth settings with distortionary taxes (Fullerton and Kim, 2008; Bovenberg and de Mooij, 1997; Hettich, 1998; Ligthart and van der Ploeg, 1994). However, these papers focus on a somewhat different set of questions than this study,[13] and follow a correspondingly different approach. For example, these studies focus on outcomes along a balanced growth path, where environmental quality is constant. In the climate change setting, this would correspond to stabilized greenhouse gas concentrations. In contrast, I study carbon taxes and fiscal policy in the short- and medium-term during the transition to balanced

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[9] In the benchmark model, the structure of preferences is such that capital income taxes are optimally set to zero after the first period.

[10] Intuitively, without a Pigouvian tax, the tradeoff between carbon-energy and climate inputs to production would be distorted. In other words, the Pigouvian tax precisely balances the reductions in the returns to investment and labor due to decreased energy inputs against the gains in productivity due to avoided climate change. For this reason, the internalization of production damages does not cause the same kind of tax interaction effect as the internalization of utility damages.

[11] In these scenarios, carbon tax revenues are used to reduce capital income tax rates. Average capital income tax rates can be lowered by 3 percentage points owing to carbon tax revenue and productivity benefits.

[12] Chiroleu-Assouline and Fodhab (2006) formally link the welfare effects of general pollution taxes to capital accumulation in an overlapping generations model with distortionary taxes. However, their focus differs from the current study; they do not consider capital income taxation and do not solve for optimal policies.

[13] For example, they study the very long run effects of pollution taxes on growth rates. I take the rate of technological progress as given.

growth. This approach builds on a wide literature that has studied environmental policy and carbon taxation in dynamic general equilibrium growth models with capital accumulation (e.g., Bovenberg and Smulders, 1996; Golosov, Hassler, Krusell, and Tsyvinski, 2011; van der Ploeg and Withagen, 2012; Briggs, 2012; Gerlagh and Liski, 2012; Heutel, 2012, etc.). The central difference in this study is that I consider taxation of carbon jointly with taxation of capital and labor from the perspective of a government that needs to both raise revenues and address climate change at the same time.

On the quantitative side, the dynamic Ramsey tax literature has attributed large welfare costs to capital income taxes (e.g., Lucas, 1990). I estimate that the welfare costs of failing to tax carbon emissions and of taxing capital income are of the same magnitude ($25 trillion, $2005 lump-sum consumption equivalent; 0.84%, permanent consumption increase).

Second, the distinction between production and utility damages relates to two sets of studies. On the theory side, Williams (2002) and Bovenberg and van der Ploeg (1994) previously established the need for this differentiation in a static setting.[14] These studies' results imply that production damages from pollution should generally be internalized with a Pigouvian tax. I provide conditions under which this result does and does not generalize to the dynamic setting. On the one hand, I show that this finding continues to hold for flow pollutants.[15] On the other hand, for accumulative pollutants such as carbon, this result does not hold if capital income is taxed. I consider two cases where the government taxes capital income, and show that Pigouvian levies on production damages are not optimal in those settings. My analysis thus theoretically extends and empirically quantifies the importance of these papers' findings for the optimal dynamic taxation of carbon emissions.

On the quantitative side, several climate-economy models aggregate all damages into pure output losses (e.g., the DICE model, Nordhaus, 2008; Golosov, Hassler, Krusell, and Tsyvinski, 2011; Leach, 2009; Cai, Judd, and Lontzek, 2012), pure utility losses (Acemoglu, Aghion, Bursztyn, and Hemous, 2011), or into market and non-market impacts (e.g., MERGE, Manne and Richels, 2004; PAGE2002, Hope, 2006; Tol, 1997). The latter is close but not identical to a separation of utility and production effects. In a setting without distortionary taxes, these separations make no difference for optimal climate policy under certain conditions (Gars, 2012). However, for an analysis of carbon taxes as a part of fiscal policy, the separation of output and utility damages is essential. To this end, I disaggregate the regional-sectoral damage estimates from the DICE model (Nordhaus, 2008) accordingly. I further add a new damage function component to capture long-term labor productivity effects of malaria exposure. I find that approximately

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[14]   In a dynamic optimal fiscal policy model, Judd (1999) provides an analogous insight for optimal levels of public spending that enter utility and production, respectively.

[15]   Flow pollutants cause damage immediately but dissipate rapidly from the environment (e.g., sulfur dioxide). In contrast, carbon dioxide accumulates and leads to damages long after it is emitted.

70% of climate change impacts from 2.5° warming affect production; 30% affect utility directly. I estimate that attributing all climate change impacts to utility (production) biases the optimal carbon tax estimate for the year 2015 downward by 20% (upward by 10%).

Third, the quantitative results build on several branches of the literature. On the one hand, several studies have employed highly detailed dynamic computable general equilibrium models to assess the welfare impacts of carbon levies in economies with distortionary taxes (e.g., Goulder, 1995; Bovenberg and Goulder, 1996; Jorgenson and Wilcoxen, 1996; Babiker, Metcalf, and Reilley, 2003; Carbone, Morgenstern, and Williams, 2012; Bernard and Vielle, 2003). Several of these studies provide welfare comparisons across different carbon tax revenue recycling regimes (e.g., reducing labor income taxes versus capital income taxes). However, these studies abstract from the environmental benefits of climate policy. On the other hand, existing estimates of optimal carbon prices from integrated assessment climate-economy models do not consider interactions with distortionary taxes.

More broadly speaking, this paper also relates to (i) studies on environmental regulation alongside pre-existing distortions that arise not from taxes but from market power (e.g., Ryan, 2012), (ii) the literature emphasizing the importance of climate-related factors for macroeconomic outcomes (e.g., Dell, Jones, and Olken, 2012), and (iii) applied studies which integrate environment-economy feedbacks from pollution into general equilibrium models with distortionary taxes (Carbone and Smith, 2008; Ballard and Medema, 1993).

How important is it to consider distortionary taxes in climate policy design? I contrast the welfare gains from imposing optimized carbon levies with the welfare gains of climate policy that ignores distortionary tax interactions.[16] Adjusting carbon taxes to account for their fiscal impacts yields a welfare gain between $600 billion and $1.5 trillion.

I structure the remainder of this paper in the following manner. Section 2 describes the core model. Section 3 provides the benchmark setting theory results. The calibration and further additions to the quantitative model are outlined in Section 4. Section 5 discusses the quantitative results. Section 6 considers extensions to a non-renewable resource setting and to preferences that are not separable in environmental quality. Finally, Section 7 offers my conclusions.

2   Model

This section describes the setup of the core version of the model. It is kept as simple as possible to maximize analytic transparency. Both subsequent theoretical extensions and the quantitative model expand upon this basic structure. A brief summary of the theoretical framework is as follows. The model essentially combines the climate-economy structure of Golosov, Hassler,

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[16]   Specifically, this policy imposes carbon taxes that would be optimal if there were no distortionary taxes.

Krusell, and Tsyvinski (GHKT) (2011) with an optimal dynamic taxation model in the Ramsey tradition (see, e.g., Chari and Kehoe, 1998). Following GHKT, I assume an infinitely-lived, representative household. An important difference to GHKT is that agents have preferences not only over consumption, but over leisure and climate change as well. As in GHKT, there are two production sectors. The aggregate final consumption-investment good is produced using capital, labor, and energy inputs. Climate change affects productivity in this sector. A carbon-based energy input is produced from capital and labor. Energy use causes greenhouse gas emissions, which accumulate and lead to climate change. Importantly, the government in GHKT does not need to raise revenues to finance public expenditures. In contrast, I incorporate an exogenous government revenue requirement, following the standard Ramsey approach. Later on, I consider exogenous social transfer spending obligations as well. The key assumption of this literature is that the government must resort to distortionary taxes because lump-sum (non-distortionary) taxes are not available for reasons outside of the model.[17] The revenues raised from Pigouvian carbon taxes are assumed to be insufficient to meet government revenue needs. Otherwise, there would be no need for distortionary taxes, and the analysis would revert to the first-best setting considered by GHKT.

Households

An infinitely-lived, representative household has preferences over consumption C_t, labor supply L_t, and a climate change variable T_t. Integrated assessment models vary in the set of climate inputs they consider (see review by Tol and Frankenhauser, 1997). I follow the common approach of using *mean global surface temperature change* over pre-industrial levels, T_t, as a sufficient statistic for climate change. Households and firms take temperature change as given. That is, climate change is an externality.[18] Households maximize lifetime utility U_0 :

$$U_0 \equiv \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, T_t) \tag{1}$$

The benchmark specification assumes that environmental quality enters preferences additively separably from consumption and leisure:

$$U(C_t, L_t, T_t) = h(C_t, L_t) + v(T_t) \tag{2}$$

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[17] One can point to several examples in recent history where governments' attempts to impose lump-sum taxes were met with intense political resistance. For example, an estimated 50% of Irish homeowners refused to pay the $133 flat-rate property tax imposed by the Irish government in January 2012 (Dalby, 2012).

[18] Alternatively, one can interpret this assumption as agents believing that their individual impact on the climate is small enough to be effectively zero.

The literature on pollution tax interactions with distortionary taxes commonly assumes weak separability. Relaxing assumption (2) does not change the main theoretical insights of this paper, which pertain to the differential and optimal dynamic taxation of output and utility climate change impacts. However, assuming separability could bias the optimal *total* carbon tax estimate upwards or downwards. This bias depends on whether temperature change is a relative complement or substitute for leisure (see, e.g., Schwartz and Repetto, 2000). Section 6.3 provides a formal discussion of this issue.

Each period, the representative household faces the following flow budget constraint:

$$C_t + \rho_t B_{t+1} + K_{t+1} \leq w_t(1 - \tau_{lt})L_t + \{1 + (r_t - \delta)(1 - \tau_{kt})\} K_t + B_t + \Pi_t \tag{3}$$

where B_{t+1} denotes one-period government bond purchases, ρ_t the price of one-period bonds, K_{t+1} the household's capital holdings in period $t+1$, w_t the gross wage, τ_{lt} linear taxes on labor income, τ_{kt} linear taxes on capital income, r_t the return on capital, δ the depreciation rate, and Π_t profits from energy production. I place several restrictions on these variables. First, capital holdings cannot be negative. The consumer's debt is bounded by some finite constant M via $B_{t+1} \geq -M$. Similarly, purchases of government debt are bounded above and below by finite constants. Finally, initial asset holdings B_0 are given.

The household's first order conditions imply that savings and labor supply decisions are governed by the standard rules, respectively:

$$\frac{U_{ct}}{U_{ct+1}} = \beta \left\{1 + (r_{t+1} - \delta)(1 - \tau_{kt+1})\right\} \tag{4}$$

$$\frac{-U_{lt}}{U_{ct}} = w_t(1 - \tau_{lt}) \tag{5}$$

where U_{it} denotes the partial derivative of utility with respect to argument i at time t. In words, the Euler equation (4) states that households equate their marginal rate of substitution between consumption in periods t and $t+1$ to the after-tax return on saving between periods t and $t+1$. Similarly, the implicit labor supply equation (5) states that agents equate their marginal rate of substitution between consumption and leisure to the after-tax return on working.

Final Goods Production

There are two production sectors: a final consumption-investment good (with input variables indexed by "1") and energy (with input variables indexed by "2"). The consumption-investment good is produced by a technology $\widetilde{F_{1t}}$ which features constant returns to scale in energy E_t, labor L_{1t}, and capital K_{1t} inputs, and satisfies the standard Inada conditions. Output Y_t further

depends on temperature change T_t and an exogenous technology parameter A_t :

$$Y_t \;=\; (1 - D(T_t)) \cdot A_{1t} \widetilde{F_{1t}}(L_{1t}, K_{1t}, E_t) \tag{6}$$
$$\;=\; F_{1t}(T_t, L_{1t}, K_{1t}, E_t) \tag{7}$$

The formulation of climate damages as fraction of output lost in (6) was pioneered by Nordhaus (1991) and is extensively used in the literature.[19],[20] A common approach is to monetize all types of damages, including ones that do not literally affect production of consumption goods (e.g., biodiversity existence value), and to subtract those costs from output as in (6). However, in a setting with distortionary taxes, differentiation between climate damages that affect physical production possibilities and those that do not is important. In the current study, formulation (6) thus represents only literal production effects of climate change. These impacts are expected to occur because the final consumption good represents an aggregate of many goods that rely on the climate as productive input, such as agriculture, fisheries, forestry products, skiing services, etc. Section 4.3 discusses this distinction in more detail.

Final goods producers chose factor inputs in competitive markets so as to equate their marginal products with their prices:

$$F_{1lt} \;=\; w_t \tag{8}$$
$$F_{1Et} \;=\; p_{Et}$$
$$F_{1kt} \;=\; r_t$$

where F_{1it} denotes the partial derivative of the final goods production function (7) with respect to input i at time t.

Energy Production

The baseline model represents energy production in a stylized manner by assuming that carbon-based energy can be produced from capital K_{2t} and labor L_{2t} inputs through a constant returns to scale technology:

$$E_t = A_{2t} F_{2t}(K_{2t}, L_{2t}) \tag{9}$$

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[19] Climate impacts can, of course, be positive for certain regions and ranges of temperature change. Indeed, the calibration of $D(T_t)$ used below follows Nordhaus and Boyer (2000) in assuming positive overall impacts from 2.5° warming for Russia. See also Tol (2002).

[20] Rezai, van der Ploeg, and Withagen (2012) study the implications of *additive* production damages.

Hotelling rents (pure profits) from energy production are then given by:

$$\Pi_t = (p_{Et} - \tau_{Et})E_t - w_t L_{2t} - r_t K_{2t} \tag{10}$$

The constant returns to scale formulation (9) assumes that carbon energy is in unlimited supply and therefore has zero Hotelling rents. As argued by GHKT (2011), this is a reasonable assumption for coal. Section 6.2 extends both the theoretical core model and the quantitative model to incorporate non-renewable carbon energy. If preferences are of a certain commonly used constant elasticity forms, the key theoretical results of the paper are unaltered by this extension. However, an important difference is that an additional term may be added into the optimal carbon tax formulation if the government cannot impose full Hotelling profits taxes. This carbon tax premium is used to indirectly tax Hotelling rents from non-renewable energy production.

The quantitative version of the model also incorporates the possibility of clean energy production (emissions abatement technologies).

I assume that both labor and capital are mobile across sectors, implying market clearing conditions:

$$L_t = L_{1t} + L_{2t} \tag{11}$$
$$K_t = K_{1t} + K_{2t}$$

This assumption is in line with GHKT (2011). Due to the 10 year time step used in the empirical model, formulation (11) is also more realistic than in an annual formulation. An important implication of (11) is that factor prices will be equated across sectors in equilibrium. Competitive energy producers thus equate marginal factor products and prices:

$$(p_{Et} - \tau_{Et})F_{2lt} = w_t \tag{12}$$
$$(p_{Et} - \tau_{Et})F_{2kt} = r_t$$

Government

As is standard in the Ramsey approach to optimal taxation, I assume that the government needs to finance an exogenously given sequence of revenue requirements $\{G_t\}_{t=0}^{\infty}$, and to pay off inherited debt B_0^G. The government can issue new, one-period bonds B_{t+1}^G and levy linear taxes on labor and capital income. In addition, the government can impose excise taxes τ_{Et} on carbon emissions E_t.[21] The consumption good serves as the untaxed numeraire. The government's flow

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[21]   Energy is denoted in units of carbon content. One ton of carbon energy thus yields one ton of carbon emissions.

budget constraint each period is given by:

$$G_t + B_t^G = \tau_{lt} w_t L_t + \tau_{Et} E_t + \tau_{kt}(r_t - \delta)K_t + \rho_t B_{t+1}^G \tag{13}$$

Market clearing requires that consumer demand and government supply for bonds be equated:

$$B_{t+1}^G = B_{t+1} \tag{14}$$

Formulation (13) builds on the standard Ramsey problem as discussed by Chari and Kehoe (1998); it specifically adds emissions taxes τ_{Et} as a new fiscal instrument.

Carbon Cycle

The only assumption placed on the carbon cycle at this stage is that temperature change T_t at time t is some function of initial carbon concentrations S_0 and all past carbon emissions:

$$T_t = \mathcal{F}\left(S_0, E_0, E_1, ..., E_t\right) \tag{15}$$

Competitive Equilibrium

Competitive equilibrium ("CE") in this economy can now be formally defined as follows:

Definition 1 *A competitive equilibrium consists of an allocation $\{C_t, L_{1t}, L_{2t}, K_{1t+1}, K_{2t+1}, E_t, T_t\}$, a set of prices $\{r_t, w_t, p_{Et}, \rho_t\}$ and a set of policies $\{\tau_{kt}, \tau_{lt}, \tau_{Et}, B_{t+1}^G\}$ such that*
   (i) the allocations solve the consumer's and the firm's problems given prices and policies,
   (ii) the government budget constraint is satisfied in every period,
   (iii) temperature change satisfies the carbon cycle constraint in every period, and
   (iii) markets clear.

The most important difference to the standard definition is the addition of the carbon cycle constraint. All variables pertaining to energy production and temperature change are also different from the standard setup in Chari and Kehoe (1998).

The Ramsey tax framework assumes that the government seeks to maximize the representative agent's lifetime utility (1) subject to the constraints of (i) feasibility and (ii) the optimizing behavior of households and firms. Note that I assume throughout that the government can commit to a sequence of tax rates at time zero.[22] The optimal allocation - the Ramsey equilibrium

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[22]   Given the potential for time inconsistency problems with regards to capital taxation in a closed economy, this is not an innocuous assumption. The availability of a commitment technology is sometimes motivated as reflecting reputational mechanisms or constitutional restrictions (see Chari and Kehoe, 1998, for a brief

- can be formally defined for a given initial level of debt B_0, an initial level of capital K_0, an initial capital tax $\overline{\tau_{k0}}$, and initial carbon concentration S_0 :

Definition 2 *A Ramsey equilibrium is the CE with the highest household lifetime utility for a given initial bond holdings B_0, initial capital K_0, initial capital tax $\overline{\tau_{k0}}$, and initial carbon concentrations S_0.*

Here, the major difference to the standard setup is the addition of initial carbon concentrations S_0. I will characterize the optimal allocations using the primal approach. By solving for optimal *allocations*, rather than for optimal tax rates, this method avoids normalization issues (see, e.g., Williams, 2001). Specifically, any good can be chosen as the untaxed numeraire. Depending on this normalization, the optimal allocation is then decentralized by a different set of prices and taxes. In other words, optimal tax rates depend on the choice of numeraire, whereas optimal allocations do not. The validity of the primal approach setup in this context requires the following proposition:

Proposition 1 *The allocations $\{C_t, L_{1t}, L_{2t}, K_{1t+1}, K_{2t+1}, E_t, T_t\}$, along with initial bond holdings B_0, initial capital K_0, initial capital tax $\overline{\tau_{k0}}$, and initial carbon concentrations S_0 in a competitive equilibrium satisfy:*

$$Y_t + (1-\delta)K_t \leq C_t + G_t + K_{t+1} \tag{RC}$$

$$T_t \geq F(S_0, E_0, E_1, ...E_t)] \tag{CCC}$$

$$E_t \leq F_{2t}(A_{Et}, K_{2t}, L_{2t}) \tag{ERC}$$

$$L_{1t} + L_{2t} \leq L_{2t} \tag{LC}$$

$$K_{1t} + K_{2t} \leq K_t \tag{KC}$$

and

$$\sum_{t=0}^{\infty}\beta^t \left[U_{ct}C_t + U_{lt}L_t\right] = U_{c0}\left[K_0\left\{1 + (F_{k0}-\delta)(1-\tau_{k_0})\right\} + B_0\right] \tag{IMP}$$

In addition, given an allocation that satisfies (RC)-(IMP), one can construct prices, debt holdings, and policies such that those allocations constitute a competitive equilibrium.

Proof: See Appendix A. This proposition and its proof differ from the standard setup in Chari and Kehoe (1998) through the addition of the energy production sector and the carbon cycle constraint. In words, Proposition 1 ensures that any allocation satisfying the six conditions

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discussion). A separate literature has explored optimal capital taxation without commitment (e.g., Klein and Rios-Rull, 2003; Benhabib and Rustichini, 1997). Extending the current setup to incorporate time inconsistency problems would be an interesting area of future research.

(RC)-(IMP) can be decentralized as a competitive equilibrium. I assume that the solution to the Ramsey problem is interior and that the planner's first order conditions are both necessary and sufficient. Formally, the government's problem is thus to maximize household lifetime utility (1) subject to the constraints (RC)-(IMP) required to ensure that the chosen allocation is both technologically feasible and consistent with competitive equilibrium:

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\begin{aligned}
\max_k \sum_{t=0}^{\infty} \beta^t & [\underbrace{U(C_t, L_t, T_t) + \phi \left[U_{ct} C_t + U_{lt} L_t \right]}_{\equiv W_t}] \\
& -\phi \left\{ U_{c0} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \tau_{k_0}) \right\} \right] \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[\left\{ A_t(T_t) \widetilde{F_{1t}}(L_{1t}, E_t, K_{1t}) \right\} + (1 - \delta) K_t - C_t - G_t - K_{t+1} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \xi_t [T_t - \mathcal{F}(S_0, E_0, E_1, ...E_t)] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{lt} \left[L_t - L_{1t} - L_{2t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{kt} \left[K_t - K_{1t} - K_{2t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \omega_t \left[F_{2t}(A_{Et}, K_{2t}, L_{2t}) - E_t \right]
\end{aligned}
\tag{16}
$$

Note that (16) follows the common approach of splitting the implementability constraint into its time-zero and lifetime summation components, and including the latter in the maximand. The key differences between (16) and the planner's problem in GHKT (2011) are (1) the implementability constraint and (2) government consumption.

Before describing the results, define the following two concepts.

The Marginal Cost of Public Funds   The marginal cost of public funds (MCF) measures the welfare cost of raising an additional dollar of government revenue. Lump-sum taxes are pure transfers: households give up \$1 to increase government revenue by \$1. Consequently, the MCF in a setting with lump-sum taxes is equal to one. In contrast, raising \$1 in revenue from distortionary taxes costs households \$1 *plus* the excess burden (or the marginal deadweight loss) created by the distortionary tax increase. Appendix B provides a summary of empirical estimates of these efficiency costs across countries from the literature. The GDP-weighted global average across tax instruments from these studies is 1.48, implying that \$0.48 cents of welfare are lost for every \$1 of government revenue raised on average. However, a caveat to pooling these estimates is that the precise definitions of the MCF, the marginal excess burden, and the

marginal deadweight loss can vary across studies (see discussions by Dahlby, 2008; Fullerton, 1991; Triest, 1990; Snow and Warren, 1996). I follow the standard approach in the literature on pollution tax interactions with other taxes of defining the MCF as follows:

Definition 3 *Let the Marginal Cost of Public Funds ("MCF") be defined as the ratio of the public marginal utility of consumption to the private marginal utility of consumption:*

$$MCF \equiv \frac{\lambda_{1t}}{U_{ct}} \tag{17}$$

The MCF thus measures the welfare cost of transferring a unit of the consumption good from households to the government. In the current setting, the MCF_t for all $t > 0$ in an optimized fiscal framework is implicitly defined by:[23]

$$MCF_t = \frac{\lambda_{1t}}{U_{ct}} = 1 + \tau_{lt}^* \left[\frac{U_{cct}C_t + U_{ct} + U_{lct}L_t}{\left\{ [U_{cct}C_t + U_{ct} + U_{lct}L_t] + \frac{1}{F_{lt}}[U_{clt}C_t + U_{lt} + U_{llt}L_t] \right\}} \right] \tag{18}$$

where τ_{lt}^* is the optimal labor income tax rate at time t. Atkinson and Stiglitz (1980) show how to relate marginal utility formulations similar to (18) to price and income elasticities in a static model by imposing additional restrictions on preferences. Similarly, numerous studies in the literature on pollution tax interactions with other taxes derive expressions for the MCF in terms of parameters and elasiticites, also in a static setting (see, e.g., Bovenberg and Goulder, 1996, Williams, 2002, also Parry and Bento, 2000, in a setting with tax deductions, etc.).[24]

Pigouvian Carbon Taxes

Definition 4 *Let the Pigouvian carbon tax be defined as the present value of marginal damages evaluated at the optimal allocation, and valued at the agent's marginal utility of consumption. More formally, the Pigouvian tax to internalize climate damages to production and utility, respectively, is given by:*

$$\text{Production damages: } \tau_{Et}^{Pigou,Y} \equiv (-1)\sum_{j=0}^{\infty} \beta^j \frac{U_{ct+j}}{U_{ct}} \left[\frac{\partial Y_{t+j}}{\partial T_{t+j}} \frac{\partial T_{t+j}}{\partial E_t} \right] \tag{19}$$

---

[23]   To derive (18), combine the first order conditions of the planner's problem with respect to C_t, L_t, and L_{1t} for $t > 0$. Rearranging and substituting in for τ_{lt} from the household's optimality conditions for labor supply (38) leads to (18).

[24]   In Appendix B.3.3, I derive an expression for the MCF along a balanced growth path for the functional forms used in the quantitative version of the model. This formulation contains preference and production parameters but cannot express the long-run MCF in closed form. This is because the long-run MCF depends on endogenous fiscal variables such as the optimal long-run labor income tax rate, which depends on the history of tax rates and government revenues collected.

$$\text{Utility damages: } \tau_{Et}^{Pigou,U} = (-1) \sum_{j=0}^{\infty} \beta^j \frac{U_{Tt+j}}{U_{ct}} \left[\frac{\partial T_{t+j}}{\partial E_t} \right] \tag{20}$$

where $\frac{\partial Y_{t+j}}{\partial T_{t+j}}$ is the marginal production loss from temperature change at time $t+j$, U_{Tt+j} denotes the marginal utility loss from temperature change at time $t + j$, and $\frac{dT_{t+j}}{dE_t}$ is the change in temperature at time $t + j$ caused by a marginal increase in today's carbon emissions dE_t. The key differentiating feature between production and utility damages is that production damages alter the economy's production possibility frontier (PPF). Conversely, utility damages affect welfare but leave production possibilities unchanged. The distinction is discussed further in Section 4.3.

The Pigouvian tax is thus defined in the standard way as the present value of marginal environmental damages, evaluated at the optimal allocation. GHKT (2011) only consider production damages and show that (19) defines the optimal carbon tax in their setting without distortionary taxes. The Pigouvian tax also equals the social cost of carbon (SCC) if the SCC is evaluated at the optimal level of emissions. Studies on the SCC differ on whether they consider the marginal impact of carbon emissions at optimal or current emissions levels (see, e.g., Pearce, 2003).

3   Theory Results

Taken together, the planner's first order conditions for consumption C_t, aggregate capital savings, K_{t+1}, and final goods production capital K_{1t} imply that, for $t > 0$,

$$\frac{W_{ct}}{W_{ct+1}} = \frac{\lambda_{1t}}{\lambda_{1t+1}} = \beta \left[F_{kt+1} + (1 - \delta) \right] \tag{21}$$

Comparison of (21) with the representative agent's Euler equation (4) demonstrates the well-known result (e.g., Atkeson, Chari, and Kehoe, 1999) that it is optimal to set effective tax capital income taxes at $t + 1$ to zero whenever:

$$\frac{\lambda_{1t}}{\lambda_{1t+1}} = \frac{U_{ct}}{U_{ct+1}} \tag{22}$$

I discuss optimal carbon tax schedules in three separate cases: If climate change affects welfare (1) only through production impacts, (2) only through direct utility losses, and (3) through both types of damages.

Case 1: Climate Change Affects Only Production

Consider first the setting where climate change affects only production. Combining the planner's first order conditions from problem (16) for emissions E_t, temperature change T_t, and the labor allocation to energy production L_{2t} implies that, for $t > 0$,

$$F_{Et} + \sum_{j=0}^{\infty} \beta^j \left[\frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial Y_{t+j}}{\partial T_{t+j}} \right] \frac{dT_{t+j}}{dE_t} = \frac{F_{1lt}}{F_{2lt}} \tag{23}$$

Expression (23) equates the social marginal costs and benefits of carbon energy input usage. The benefits consist of the marginal product of energy in final goods production, F_{Et}, minus the sum of future production losses $\left(\frac{\partial Y_{t+j}}{\partial T_{t+j}} \right)$ due to the additional climate change resulting from time t carbon emissions $\left(\frac{dT_{t+j}}{dE_t} \right)$. Note that the planner values future production losses at the public value of output in each time period, λ_{1t+j}. The private marginal cost of carbon energy is simply the production cost expressed in units of the final consumption good $\left(\frac{F_{1lt}}{F_{2lt}} \right)$.

What carbon tax τ_{Et} can decentralize (23)? Substitute for equilibrium prices in (23) based on the producers' first order conditions (8) and (12). Rearranging the resulting terms immediately yields the following result.

The carbon price in period $t > 0$ that decentralizes the optimal allocation, provided that all other prices and taxes are set appropriately, is implicitly defined by:

$$\tau_{Et}^* = (-1) \sum_{j=0}^{\infty} \beta^j \left[\frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial Y_{t+j}}{\partial T_{t+j}} \right] \frac{dT_{t+j}}{dE_t} \tag{24}$$

Expression (24) captures the social cost of carbon energy usage. Intuitively, this value embodies the difference between the social and private marginal cost of carbon energy, and thus represents the optimal carbon tax. The first result follows immediately.

Proposition 2 *If the government optimally sets capital income taxes to zero from period $t + 1$ onwards, then the optimal carbon tax to internalize production damages at time $t > 0$ is the Pigouvian tax.*

Proof. First, for all $j \geq 1$, multiply the $t + j^{th}$ term in the sum of (24) by:

$$\left(\prod_{m=1}^{j-1} \frac{\lambda_{1t+m}}{\lambda_{1t+m}} \right) = 1$$

Each term $\frac{\lambda_{1t+j}}{\lambda_{1t}}$ can then be rearranged to equal $\left(\frac{\lambda_{1t+j}}{\lambda_{1t+j-1}} \frac{\lambda_{1t+j-1}}{\lambda_{1t-2}} ... \frac{\lambda_{1t+1}}{\lambda_{1t}} \right)$.

Second, note that the optimality of zero capital income taxes from period $t+1$ onwards implies that condition (22) must be satisfied for all $t+j$, $j \geq 1$. That is, for all $j \geq 1$,

$$\frac{\lambda_{1t+j}}{\lambda_{1t+j-1}} = \frac{U_{ct+j}}{U_{ct+j-1}} \tag{25}$$

Third, repeatedly use (25) to substitute out for all $\frac{\lambda_{1t+j}}{\lambda_{1t+j-1}}$ terms in the sum on the right-hand side of (24), which then becomes:

$$\tau_{Et}^* = (-1) \sum_{j=0}^{\infty} \beta^j \left[\frac{U_{ct+j}}{U_{ct}} \frac{\partial Y_{t+j}}{\partial T_{t+j}} \right] \frac{dT_{t+j}}{dE_t} = \tau_{Et}^{Pigou,Y} \tag{26}$$

Finally, comparison with the definition of the Pigouvian tax (19) demonstrates the desired result. Note that no manipulation of the $j = 0$ term in the summation is necessary because $(\lambda_{1t}/\lambda_{1t} = U_{ct}/U_{ct} = 1)$ regardless of whether condition (22) is satisfied. ∎

The intuition for this result is twofold, and can be summarized as follows. First, the climate is an asset used in production (e.g., of agriculture), analogous to physical capital. Second, pricing carbon emissions at less-than-Pigouvian rates is conceptually equivalent to taxing climate capital investments. Consequently, the economic factors that make it desirable for the government to leave households' physical capital investments undistorted likewise make it desirable to leave investments in environmental capital undistorted. This requires precisely a Pigouvian tax.

To make these points more concrete, briefly consider a simplified two period version of the model. The marginal rate of transformation (MRT) between consumption in the two periods based on investment in physical capital is given by:

$$MRT_{0,1}^K = \frac{\text{Give up 1 unit of } C_0 \text{ to invest in capital}}{\text{Get } (F_{k1}+(1-\delta)) \text{ units of } C_1 \text{ tomorrow:}} = \frac{-1}{F_{k1} + 1 - \delta}$$

An undistorted intertemporal margin requires that this $MRT_{0,1}^K$ be equated with the household's marginal rate of substitution (MRS) between consumption in the two periods:

$$MRS_{0,1} = MRT_{0,1}^K \tag{27}$$

$$\frac{\beta U_{c1}}{U_{c0}} = \frac{1}{F_{k1} + 1 - \delta} \tag{28}$$

Implementing the allocation (28) requires a zero effective capital income tax. However, the key issue in this economy is that there is an additional technology for converting C_0 into C_1:

investments in climate capital.[25] Specifically, assume initial period carbon emissions E_0 are reduced by one unit. In terms of the initial period consumption good, this will create a loss of F_{E0}, the marginal product of energy. However, it will also save marginal energy production costs MC. The net loss of C_0 associated with the emissions reduction is thus $F_{E0} - MC$.[26] The return on this investment is avoided output losses from climate change in the next period. Specifically, the gain in terms of C_1 is the additional output from marginally lower temperature change $(\partial Y_1/\partial T_1)$, multiplied by the actual decrease in temperature change achieved by the reduction in E_0 $(\partial T_1/\partial E_0)$. In sum, the MRT based on investments in climate capital is given by:

$$MRT_{0,1}^{\text{Climate}} = \frac{\text{Reduce } E_0 \text{ by 1 unit} \rightarrow \text{Give up } F_{E0} - MC \text{ units of } C_0}{\text{Get } (\partial Y_1/\partial T_1)(\partial T_1/\partial E_0) \text{ units of } C_1} = \frac{F_{E0} - MC}{(\partial Y_1/\partial T_1)(\partial T_1/\partial E_0)}$$

Equating the household's MRS with this second MRT yields:

$$MRS_{0,1} \;=\; MRT_{0,1}^{\text{Climate}} \tag{29}$$

$$\frac{\beta U_{c1}}{U_{c0}} \;=\; \frac{F_{E0} - MC}{(\partial Y_1/\partial T_1)(\partial T_1/\partial E_0)} \tag{30}$$

What carbon tax decentralizes (30)? Multiplying both sides by $(\partial Y_1/\partial T_1)(\partial T_1/\partial E_0)$ immediately demonstrates that an undistorted intertemporal margin for climate capital investments requires precisely a Pigouvian tax on carbon:

$$\frac{\beta U_{c1}}{U_{c0}} \left(\frac{\partial Y_1}{\partial T_1} \frac{\partial T_1}{\partial E_0} \right) = F_{E0} - MC = \tau_{E0}^{Pigou}$$

Here, the second equality follows because (i) competitive factor pricing implies that $F_{E0} = p_{E0}$ in equilibrium (see (8)), and (ii) the energy sector produces carbon up until the point where $(p_{Et} - \tau_{Et}) = MC$ (see (12)).

The literature on optimal dynamic Ramsey taxation has found that capital income taxes are undesirable in wide range of models and settings (see, e.g., Chamley, 1985; Judd, 1986; Atkeson, Chari, and Kehoe, 1999; Acemoglu, Golosov, and Tsyvinski, 2011). A number of studies have explored the implications of this result for human capital taxation (Judd, 1999; Jones, Manuelli, and Rossi, 1993, 1997). Proposition 3 demonstrates that the logic against capital income taxes further extends to environmental capital.

In reality, most countries do impose capital income taxes (Piketty and Saez, 2012; Mankiw,

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[25] The general idea that investment in natural capital should be considered as part of a portfolio problem along with physical capital has been formalized in many previous studies (e.g., Bovenberg and Smulders, 1996; Fullerton and Kim, 2008; see also Nordhaus, 2010, discussing the climate as natural capital stock).

[26] This illustration assumes no contemporaneous climate change impacts.

Weinzierl, and Yagan, 2009.). A natural follow-up question to Proposition 3 is thus: What is the optimal structure of carbon taxes in an economy where capital taxes are not zero? Perhaps surprisingly, the answer can depend on the underlying reason *why* capital taxes are positive. In Section 6.1, I analyze two extensions of the core model that involve positive capital income taxes and affect carbon tax schedules differently.

First, with an upper bound on capital income tax rates, the government sets capital income taxes at this upper bound for a finite number of periods and eventually decreases them to zero. In this setting, carbon taxes to internalize output damages are lower than Pigouvian rates for as long as capital income taxes remain positive.

Second, with an exogenous constraint that capital income tax rates be fixed at some positive level, carbon taxes to internalize output damages may be adjusted upwards or downwards relative to Pigouvian rates. The adjustment depends in part on the impact of output damages on the tightness and direction with which the capital income tax constraint binds.

There are other modifications of the basic Ramsey setup and fundamentally different models of taxation that imply the desirability of capital income taxes (see, e.g., Golosov, Kocherlakota, and Tsyvinski, 2003; Erosa and Gervais, 2002, etc.). Integrating climate capital into these models and exploring optimal carbon taxes in those frameworks is beyond the scope of this study but an interesting area for future research.[27] In summary, the results of this section suggest that the reasons against capital income taxation brought forth by the benchmark Ramsey model extend to environmental capital, and imply the optimality of Pigouvian taxes to internalize production losses from climate change.

Case 2: Climate Change Affects Only Utility

Consider now climate change impacts that affect preferences but do not alter production possibilities. For example, biodiversity existence value losses from species extinctions or health impacts on non-working populations affect human welfare but not productivity.

Proposition 3 *The optimal carbon tax to internalize utility damages in period $t > 0$ is implicitly defined by:*

$$\tau_{Et}^* = \frac{\tau_{Et}^{Pigou,U}}{MCF_t} \tag{31}$$

where MCF_t is the contemporaneous marginal cost of public funds as defined in (17).

Proof. Proceeding analogously to Case 1, first combine the planner's first order conditions for emissions E_t, temperature change T_t, and the labor allocation to energy production L_{2t}. For

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[27] For example, Cremer, Gahvari, and Ladoux (2001) study optimal pollution taxation in a Mirrleesian taxation model, where distortions arise due to informational frictions. Their study focuses on a static setting. It would thus be interesting to extend their work to the dynamic setting.

periods $t > 0$, this yields:

$$F_{Et} + \sum_{j=0}^{\infty} \beta^j \left[\frac{U_{Tt+j}}{\lambda_{1t}} \right] \frac{dT_{t+j}}{dE_t} = \frac{F_{1lt}}{F_{2lt}} \tag{32}$$

Next, invoke competitive equilibrium prices based on (8) and (12) to find the implicitly defined optimal tax:

$$\tau_{Et}^* = \sum_{j=0}^{\infty} \beta^j \left[\frac{U_{Tt+j}}{\lambda_{1t}} \right] \frac{dT_{t+j}}{dE_t} \tag{33}$$

Finally, multiply the right hand side of (33) by $\frac{U_{ct}}{U_{ct}}$. Applying the definition of the marginal cost of public funds (17) completes the proof. $\blacksquare$

While formulation (31) only defines optimal carbon taxes implicitly, it demonstrate that optimal carbon levies are *below* Pigouvian rates if the marginal cost of public funds is greater than one.[28] That is, the optimal carbon tax does not internalize utility losses from climate change fully. In contrast, as discussed in Case 1, the optimal tax internalizes *output losses* fully in a wide range of model structures. I will discuss the intuition for this difference from two perspectives: first, the difference in tax interactions, and second, optimal commodity taxation theory.

When climate change affects only utility, imposing a carbon tax to reduce global warming does not yield any production benefits. To the contrary, carbon taxes decrease the returns to labor. This is because carbon taxes increase the cost of energy inputs and thus the cost of producing the consumption-investment good. As a result, carbon taxes can increase the costs of consumption relative to leisure, and hence decrease the returns to labor.[29] Importantly, carbon taxes can thus exacerbate the effects of labor income taxes, which alter labor supply decisions by lowering the after-tax return to labor.[30] The MCF_t measures the marginal welfare cost of taxation. The optimal climate policy thus discounts utility damages by the MCF_t to account for carbon tax interactions with other taxes. Intuitively, these interactions increase the cost of providing the public consumption good of environmental quality.

In contrast, when climate change affects production possibilities, carbon taxes are levied specifically to increase production efficiency. That is, the environmental benefits of carbon taxes can offset the increases in production costs resulting from higher energy prices. As a result, the labor tax interaction effect does not arise with output damages, as long as carbon levies are set appropriately. Climate policy must weigh output losses due to reduced energy usage in the

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[28] This statement pertains to the marginal cost of funds evaluated at the carbon tax-inclusive allocation. Specifically, the optimal carbon levy is less than the Pigouvian tax evaluated at the optimal allocation with distortionary taxes.

[29] Higher energy prices could also decrease the marginal products of labor and capital directly through their effects on energy input use and the subsequent general equilibrium adjustments.

[30] This is the tax interaction effect that has been most extensively studied in the literature (see review by Bovenberg and Goulder, 2002).

present against output gains due to avoided climate change in the future. As discussed above, the Pigouvian tax (19) precisely balances these costs and benefits if there are no intertemporal distortions (i.e., no capital income taxes).

One can also explain the difference between Case 1 and Case 2 by appealing to optimal commodity taxation theory. Utility damages reflect the value of the climate as a final consumption good (e.g., existence value for biodiversity). Conversely, output damages reflect the value of the climate as an input to production (e.g., in agriculture). The intermediate goods taxation theorem states that it is preferable to distort consumption of final goods rather than usage of intermediate inputs. This is because taxing the latter leads to violations of aggregate production efficiency (Diamond and Mirrlees, 1971). With utility damages, setting $\tau_{Et} < \tau_{Et}^{Pigou}$ distorts consumption of the climate good. With output damages, setting $\tau_{Et} < \tau_{Et}^{Pigou}$ distorts usage of the climate input. As a result, setting $\tau_{Et} < \tau_{Et}^{Pigou}$ to account for tax interactions is desirable in the case of utility damages, but commonly undesirable in the case of output damages.

The static version of (31), ($\tau_E^* = \tau_E^{Pigou}/MCF$), is a classic formulation in the literature on pollution taxes and distortionary taxes (e.g., Bovenberg and van der Ploeg, 1994; Bovenberg and Goulder, 1996, etc.) Proposition 4 thus provides a generalization of this formulation to carbon taxation in a dynamic setting with capital.

Case 3: Climate Change Affects Both Production and Utility

In the realistic case that climate change affects both production and utility, the optimal carbon tax for $t > 0$ is implicitly defined by:

$$\tau_{Et}^* = \left[\frac{\tau_{Et}^{Pigou,U}}{MCF_t} \right] - \sum_{j=0}^{\infty} \beta^j \left[\frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial Y_{t+j}}{\partial T_{t+j}} \right] \frac{dT_{t+j}}{dE_t} \tag{34}$$

The derivation of (34) is analogous to the procedure outlined for Case 1 and Case 2 above.

Remark 5 *If preferences are of either commonly used constant elasticity form,*

$$U(C_t, L_t, T_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \vartheta(L_t) + v(T_t) \tag{35}$$

$$U(C_t, L_t, T_t) = \frac{(C_t L_t^{-\gamma})^{1-\sigma}}{1-\sigma} + v(T_t) \tag{36}$$

then the optimal carbon tax for period $t > 0$ is implicitly defined by:

$$\tau_{Et}^* = \tau_{Et}^{Pigou,Y} + \frac{\tau_{Et}^{Pigou,U}}{MCF_t} \tag{37}$$

21

This result follows from (34) and the observation that preferences of the form (35) or (36) imply that $\frac{\lambda_{1t+1}}{\lambda_t} = \frac{U_{ct+1}}{U_{ct}}$ for all t.

Expressions (34) and (37) demonstrate that it is critical to distinguish between climate change impacts on production and on utility. These types of damages are internalized differently in a setting with distortionary taxes. In the literature on pollution tax interactions with other taxes, many studies assume that environmental quality affects only utility (see review in Bovenberg and Goulder, 2002). However, a few studies have previously emphasized the need for this damage type distinction (Williams, 2002; Bovenberg and van der Ploeg, 1994) in a static setting, and have derived analogous formulations to (37). Expressions (34) and (37) thus extend these studies' finding to the dynamic taxation of carbon.

Many climate-economy models aggregate all damages into pure output losses (e.g., the DICE/RICE models, Nordhaus, 2010; Golosov, Hassler, Krusell, and Tsyvinski, 2011; Leach, 2009), pure utility losses (Acemoglu, Aghion, Bursztyn, and Hemous, 2011), or into market and non-market impacts (e.g., MERGE, Manne and Richels, 2004; PAGE2002, Hope, 2006; Tol, 1995). The latter is similar but not the same as a disaggregation into utility and production damages. On the basis of these theoretical results, I propose an alternative representation of climate change impacts that accounts separately for production and utility damages, as discussed in detail in Section (4.3).

4 Calibration of the COMET Model

4.1 Model Overview

The **C**limate **O**ptimization **M**odel of the **E**conomy and **T**axation (COMET) outlined above could be combined with a range of integrated assessment climate-economy models. Given its status as benchmark in the literature, I choose the DICE (Dynamic Integrated Climate Economy) model (Nordhaus, 2008) as a baseline. Table 1 provides an overview of the quantitative model components, delineating which features have been (i) adopted directly from DICE, (ii) adapted for the purposes of the COMET, or (iii) newly created for COMET:

Adopted from DICE	Adapted for COMET	New for COMET
Carbon cycle	Damage function	Energy production
Abatement costs	Preferences	Government expenditures
Productivity growth	Final goods production	Tax policy
Population growth		

Table 1: Overlap of DICE and COMET Model Features

The COMET model assumes a global planner who is looking for the optimal carbon tax to maximize global welfare. In reality, taxation is a national policy matter. How does the COMET model relate to individual countries with different fiscal policies and constraints? The welfare costs from distortionary tax interactions create a wedge between the social and private marginal costs of carbon emissions reductions (or marginal abatement costs, MAC). Bovenberg and Goulder (1996) essentially estimate this wedge for the U.S. economy (gross of environmental benefits). Both private and social abatement cost structures differ across countries due to variations in technology, industry composition, tax systems, etc. Babiker, Metcalf, and Reilley (2003) demonstrate heterogeneous non-environmental welfare impacts of carbon taxes and revenue recycling schemes across countries; see also Bernard and Veille (2003). This situation is arguably analogous to the textbook case of optimal pollution control across firms with heterogeneous emissions reduction costs (e.g., Gruber, 2005). As is well-known, the efficiency-maximizing policy equates *aggregate* marginal abatement costs with marginal emissions reduction benefits. Intuitively, the DICE model (Nordhaus, 2010) performs this tradeoff for private abatement costs. In contrast, the current model seeks to weigh a measure of aggregate social or tax-interaction-inclusive marginal abatement costs against climate protection benefits to solve for the global optimal policy. My aggregate tax interaction cost measure has notable shortcomings in that it does not incorporate factors such as trade interactions or policy implementation constraints. In addition, the measure is not based on a direct horizontal aggregation of country-level tax interaction cost curves. Rather, it uses GDP-weighted global average measures of government revenue needs, government transfers, and tax rates to capture representative global distortions. The benefit of this approach is that it provides comparatively transparent insights to the main question of how optimal carbon price estimates are affected by distortionary tax interactions in a well-known framework, and to the underlying mechanisms at play. The central quantitative finding is that optimal carbon tax rates are $20 - 35\%$ lower in a setting with distortionary taxes, compared to the setting with lump-sum taxes generally considered in the IAM literature. As discussed further in the conclusion, the integration of distortionary taxes into multi-region IAMs is thus arguably an interesting area for future research.

4.2 Carbon Cycle and Climate Model

The carbon cycle is taken directly from the 2010 DICE model (Nordhaus, 2010). It is represented by three carbon reservoirs: the atmosphere (S_t), the upper oceans and biosphere (S_t^{Up}), and the deep oceans (S_t^{Lo}). Endogenous industrial carbon emissions E_t and exogenous land-based emissions E_t^{Land} first enter the atmosphere, and subsequently begin to be absorbed by the upper oceans and biosphere. There is two-way mixing between adjacent carbon reservoirs, and the

corresponding evolution of concentrations can be represented as:

$$
\begin{pmatrix} S_t^{At} \\ S_t^{Up} \\ S_t^{Lo} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{21} & 0 \\ \phi_{12} & \phi_{22} & \phi_{32} \\ 0 & \phi_{23} & \phi_{33} \end{pmatrix} \begin{pmatrix} S_{t-1}^{At} \\ S_{t-1}^{Up} \\ S_{t-1}^{Lo} \end{pmatrix} + \begin{pmatrix} E_t + E_t^{Land} \\ 0 \\ 0 \end{pmatrix}
$$

Changes in atmospheric carbon concentrations lead to increases in radiative forcings F_t. Loosely speaking, radiative forcings measure the net change in the earth's radiation energy balance measured in watts/m^2. Along with other, exogenous radiative forcings F_t^X, this effect is captured by:[31]

$$
F_t = \eta \{ \ln \left(\frac{S_t}{S_{1750}} \right) / \ln(2) \} + F_t^X
$$

Finally, increased radiative forcing leads to atmospheric temperature change. The DICE carbon cycle keeps track of both atmospheric and lower ocean temperature change, which evolve according to:

$$
\begin{pmatrix} T_t^{At} \\ T_t^{Lo} \end{pmatrix} = \begin{pmatrix} (1 - \xi_1 \xi_2 - \xi_1 \xi_3) & \xi_1 \xi_3 \\ (1 - \xi_4) & \xi_4 \end{pmatrix} \begin{pmatrix} T_{t-1}^{At} \\ T_{t-1}^{Lo} \end{pmatrix} + \begin{pmatrix} \xi_1 F_t \\ 0 \end{pmatrix}
$$

The parameters of the carbon cycle and climate model are set such that the equilibrium temperature change associated with a doubling of carbon dioxide concentrations - the climate sensitivity - is $3.2°C$.

4.3 Damages

The theoretical results demonstrate that it is essential to account separately for production and utility damages from climate change in an environment with distortionary taxes. In this section, I first briefly survey different approaches to modeling climate damages that have been taken in the literature, and then describe my approach.

Many integrated assessment models aggregate all climate damages into pure production losses (e.g., Nordhaus, 2008; Golosov, Hassler, Krusell, and Tsyvinski, 2011). Some models aggregate all climate damages into utility losses (e.g., Acemoglu, Aghion, Bursztyn, and Hemous, 2011). Other studies classify damages into market and non-market impacts, as discussed below. In a setting without distortionary taxes, these separations make no difference for climate policy under certain conditions (Gars, 2012).

A number of authors differentiate climate damages into categories of economic and non-economic (*Page2002*, Hope, 2006), tangible and intangible (*Fund*, Tol, 1995, 1997), or market

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[31]   Examples of sources of exogenous forcings include aerosols, ozone, and chloroflourocarbons (Nordhaus, 2008).

and non-market (*Merge*, Manne and Richels, 2004). These categorizations essentially distinguish damages with direct market impacts from those "for which there are no market values" or that "are difficult to monetize" (Tol, 1994; Manne and Richels, 2004; Plambeck, Hope, and Anderson, 1997). This distinction is almost identical to production and utility damages. However, there are several differences between the categorizations of market/non-market damages and production/utility damages.

First, some climate damages may be difficult to monetize *ex-ante*, but their *ex-post* impacts entail significant shifts of the production possibility frontier. For example, Manne and Richels (2006) mention a shutdown of the North Atlantic thermohaline circulation as an example of non-market damages. However, such an event would assuredly affect productivity (Link and Tol, 2004), and should not be categorized as pure utility impact.

Second, while human health is classified as non-market good in the three models cited above, health impacts can alter production possibilities by affecting the global labor force time endowment, labor productivity, and health expenditures. I discuss this issue in further detail below.

Third, utility damages could affect equilibrium prices if climate change did not enter preferences separably. That is, by moving agents' offer curves, utility damages can change equilibrium prices without altering the production possibility frontier. The COMET model currently abstracts from these price impacts by assuming separable preferences of the form (41). However, this issue is discussed in further detail in Section 6.3.

Table 2 showcases estimates of total market and non-market climate impacts from the literature:

		Scenario	Damages (% of GDP)		
Model	Region	ΔT ($C°$)	Market	Non-Market	%Market
PAGE2002	European Union (mean)	2.5°	0.5%	0.73%	40.6%
MERGE2004	Wealthy nations	2.5°	0.25%	2%	11%
	Per capita income of $25,00		0.5%	1%	33%
	Per capita income of $5,000		0.5%	~ 0%	~ 100%
FUND (1995)	Global aggregate	$[2 \times CO_2]$	0.31%[†]	1.59%	16%
ICAM 2.5	Developed	$[2 \times CO_2]$	0.5%[‡]	2%	20%
	Developing		2.5%	0.5%	83%
Nordhaus (1994)	Expert Survey (mean)	3°			62.4%

[†]Computed using Tol's (1995) description of the fraction of damages in each sector considered "tangible" along with the damage estimates from Appendix Table A1.

[‡]Figures taken from Tol and Frankenhauser (1997)

Table 2: Differentiated Damage Estimates

In order to maintain close comparability with the DICE model, I mainly derive estimates of production and utility damages by splitting and re-aggregating the regional-sectoral[32] damage estimates underlying the DICE/RICE models into these two categories (Nordhaus, 2007; Nordhaus and Boyer, 2000). Table 3 shows the proposed classification scheme:

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[32] The distinct regions represented are: the United States, Western Europe, Russia, Eastern Europe/former Soviet Union, Japan, China, India, Middle East, Sub-Saharan Africa, Latin America, other Asian countries, and other high income countries.

Impact Category	Classification
Agriculture	Production
Other vulnerable markets (energy services, forestry production, etc.)	Production
Sea-level rise coastal impacts	Production
Amenity value	Utility
Ecosystems	Utility
Human (re)settlement	Utility
Catastrophic damages	Mixed
Health	Mixed

Table 3: Climate Damage Categorization

For catastrophic damages, the figures underlying the DICE model are based on expected damages from catastrophic outcomes. These outcomes are defined as equivalent to a permanent income loss of 30% of global GDP. However, this loss represents both literal output losses and accounts for disutility of non-production damages. Climate "tipping point" impacts, such as a shutdown of the North Atlantic thermohaline circulation, or changes in the Indian summer monsoon would likely affect ecosystems, human health, and human settlements in addition to causing production impacts (see, e.g., Lenton et al., 2008; IPCC Working Group II, 2007). For each region, I thus split catastrophic damages into production and utility components according to the region's share of non-catastrophic impacts affecting production and utility, respectively. There are two noteworthy technical points. First, I use the absolute value of total non-catastrophic impacts for each region in this calculation. This is to avoid miscalculating the relative importance of production or utility damages in regions where positive and negative impacts of 2.5° warming cancel out to a certain extent.[33] Second, I exclude climate amenity values from the calculation of production and utility shares of catastrophic damages, as amenity value changes do not appear to be an important component of damages associated with catastrophic climate change.

Health Impacts

The health impacts of climate change affect welfare in at least four key ways: mortality, morbidity, labor productivity, and health expenditures. The integrated assessment climate-economy models

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[33] For example, in Russia, total production impacts of 2.5° warming are projected to be positive (negative damages), whereas health, ecosystem, and human settlement impacts are expected to be negative (positive damages). *Total non-catastrophic damages* for Russia are thus less than utility damages, implying a share of catastrophic utility impacts greater than 100%. Consideration of the absolute value share of climate change impacts in each category thus arguably represents the relative importance of either category more accurately.

cited above focus on mortality and morbidity impacts of climate-sensitive diseases. A common approach is to value lost life years in accordance with the value of statistical life (VSL) literature (e.g., DICE, Nordhaus, 2008; FUND 3.5, Anthoff and Tol, 2010). For the models that separate market and non-market damages, these losses are then generally classified as non-market impacts (Page2002, Hope, 2006; Fund, Tol, 1995, 1997; Merge, Manne and Richels, 2004).

The consideration of general equilibrium effects from distortionary taxes and labor supply in this study complicates the appropriate modeling of health impacts considerably. In particular, treating statistical losses of life as a consumption good of the representative agent misses the labor market impacts of changes in the world's time endowment due to morbidity and mortality effects. However, changes in agents' time endowments have both leisure and labor impacts, depending on the relevant elasticities. Williams (2002) provides a detailed theoretical treatment of these issues. Capturing all the details accurately in the climate change setting would likely require a dynamic heterogenous agent model with endogenous probabilities of death and disease as well as general equilibrium wage effects at the regional level. Such a detailed treatment is beyond the scope of this study. I nonetheless seek to capture and differentiate labor effects and value of statistical life losses in a simplified framework, as discussed below. In addition, I compute a new damage function component to incorporate long-term labor productivity impacts from malaria exposure. Labor productivity impacts have not generally been included in standard integrated assessment models (Tol, 2011). While this paper considers only one channel through which climate change can affect labor productivity, I view this as an important area to explore, as pollution impacts on labor productivity and on mortality/morbidity figure into the optimal tax formulation differently (Williams, 2002). The remainder of this subsection discusses the three categories of health impacts included in COMET in further detail.

First, the framework for assigning years of life lost (YLL) to *labor-production losses* is as follows. Individual households chose to supply fraction l_t of their productive time endowment in decade t, ω_t. Following Jones, Manuelli, and Rossi (1993), 60.4% of time is assumed to be available for productive purposes (14.5 hours per day). The basic normalization of the model sets $\omega_t = 1$. The aggregate productive time endowment per decade is thus $\Omega_t = N_t \omega_t$, and aggregate labor supply is $L_t = l_t \Omega_t$. The final goods production technology is Cobb-Douglas (see below in (46)) and can thus be written as:

$$Y_t = (1 - \widehat{D}(T_t))A_t \cdot K_t^\alpha E_t^v [l_t \cdot \Omega_t]^{1-\alpha-v} \tag{38}$$

where $\widehat{D}(.)$ represents production climate damages gross of labor health impacts. Let $\xi(T_t)$ denote the fraction of the aggregate productive time endowment lost due to climate change-induced YLLs from T_t degrees of warming. Output net of health-labor losses Y_t' is thus given

by:

$$Y_t' = (1 - \widehat{D}(T_t))A_t \cdot K_t^\alpha E_t^v [l_t \cdot \Omega_t (1 - \xi(T_t))]^{1-\alpha-v}$$
$$= (1 - \xi(T_t))^{1-\alpha-v} \cdot Y_t \tag{39}$$

I use the regional YLLs implied by the DICE model (Nordhaus, 2007) to calculate production damages from health-related labor time losses according to (39).[34] Equilibrium global labor supply remains fully endogenous. However, to maintain a given level of output Y_t, labor supply l_t has to be increased to compensate for the loss in the time endowment $\xi(T_t)$. Intuitively, if a household member falls sick to malaria, the other household members have to increase labor supply to maintain a given level of income. Note that I do not adjust the population level N_t to account for deaths, since doing so would *decrease* the welfare weight given to the generation alive at time t.

Second, the *non-labor component of YLLs* is valued as a consumption good (utility loss). The specific measure is two times per capita income per YLL, following (Nordhaus and Boyer, 2002).[35,36] To avoid double-counting, I discount YLLs by the baseline share of time spent on leisure (77%, see Appendix C).

Third, *labor productivity* impact estimates included in COMET account for long-term effects of malaria exposure. Malaria is one of the most climate-sensitive diseases (WHO, 2009). There is growing empirical evidence on the long-term effects of malaria exposure on labor productivity (Bleakley, 2003, 2010; see also discussion in Gollin and Zimmerman, 2007). A central underlying

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[34] I also considered estimates from a report from the World Health Organization (WHO) on climate change and health (McMichael et al. 2004). Unfortunately, the report only provides direct estimates of disease-adjusted life years lost (DALYs) for the year 2000, based on a backwards-extrapolation of their model output. First, I thus extrapolate the temperature change implied by their climate scenarios for 2000. Second, I calibrate a quadratic temperature-based damage function given their estimates, and project it forward to 2.5° warming. The 2010-DICE values yield a GDP-weighted global loss for *utility* health impacts of 0.09% of GDP for $2.5°C$. In contrast, the value calculated based on the WHO estimates is significantly larger at 0.47% of GDP. Using this estimate would affect my quantitative results, and would increase the share of utility damages to around 50%. The WHO estimates are likely on the high side, however. For example, they assume zero adaptation to increased malaria risk, even with rising incomes.

[35] It is important to note that VSL values pertain to the value of a *statistical* life year loss, and do not "put a price tag" on the lives of any actual, specific human beings.

[36] Alternatively, one could value the utility/non-labor component of YLLs at the price of leisure. Jorgenson, Goettle, Hurd, Smith, and Mills (2004) essentially follow this approach. They integrate the health impacts from thermal stress and tropospheric ozone due to climate change into a dynamic general equilibrium model of the U.S. economy. More specifically, the authors decrease agents' time endowments in accordance with the health impacts, and evaluate the welfare costs of the associated changes in leisure and consumption. Their resulting estimate of the value of a statistical life is at the low end of the VSL literature, and considerably below the U.S. Environmental Protection Agency's standard value. As such, the authors note that there is likely a willingness-to-pay premium to avoid statistical loss of life above and beyond the direct value of consumption and leisure losses. The authors further point to the possibility of adding a VSL premium to market-based damage estimates.

mechanism is anemia, which has been shown to significantly impair labor productivity, including in large-scale field-experiments (Duncan et al., 2004). Lucas (2010) finds evidence of significant increases in educational attainment due to malaria eradication. I use Bleakley's (2003) estimate that a malarious childhood decreases adult wages by 15%,[37] along with Tol's (2008) estimates of climate change-induced increases in malaria morbidity, and World Bank data on baseline malaria prevalence[38] to calculate GDP-weighted labor productivity losses from 2.5° warming. At the global level, these impacts are small: I find an estimated decrease in total factor productivity of 0.0105%. However, in Sub-Saharan Africa, malaria-induced total factor productivity losses from 2.5° warming alone are predicted to be around 0.33%. These impacts are added to the production damages as outlined in Table 3.

Production vs. Utility Damages: Results

The base year GDP-weighted estimates for production and utility damages are as follows:

$$\text{Total damages from } 2.5° \text{ warming} \quad = \quad 1.44\% \text{ of output} \tag{40}$$

$$\text{Total } \textbf{production damages} \quad : \quad 1.06\% \text{ of output}$$

$$\text{Total } \textbf{direct utility damages} \quad : \quad 0.37\% \text{ of output}$$

$$\text{Share of output damages} \quad : \quad 74\%$$

The COMET adopts the functional form of the output damage function $D(T_t)$ from the DICE model. However, the damage coefficient θ_1 is calibrated based on (40) such that output losses from 2.5° temperature change equal 1.06% of output rather than 1.44% of output:

$$
\begin{aligned}
(1 - D_t(T_t)) &= \frac{1}{1 + \theta_1 T_t^2} \\
1 - 0.0106 &= \frac{1}{1 + \theta_1 (2.5)^2} \\
\theta_1 &= 0.00172
\end{aligned}
$$

The calibration of utility damages is discussed in the following section on preferences.

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[37] Gollin and Zimmerman (2007) use a slightly lower value of 10%. However, in more recent work, Bleakley (2007) finds evidence for impacts considerably larger than 15%.

[38] World Bank *World Development Indicators*, "Notified cases of malaria (per 100,000 people)," for all available countries, year 2008.

4.4 Preferences

In the DICE model, the representative agent has preferences over consumption. The current setup adds preferences over leisure and temperature change. The essential traits of a utility function for the current setting is that it be: (1) consistent with a balanced growth path, (2) able to match the intertemporal elasticity of substitution (IES) from the DICE model, and (3) possible to calibrate to a desired range of Frisch elasticity of labor supply values, given benchmark labor supply estimates. The utility function chosen to satisfy these criteria is:

$$U(c_t, l_t, T_t) = \left\{ \frac{[c_t \cdot (1 - \phi l_t)^\gamma]^{1-\sigma}}{1-\sigma} \right\} + \alpha_0 (T_t)^2 \tag{41}$$

where c_t and l_t are individual-level consumption and labor supply ($c_t = C_t/N_t$, where N_t is the population at time t). Specification (41) is based on King-Plosser-Rebelo (KRB) preferences (King, Plosser, and Rebelo, 2001), with two modifications. The first is the addition of preferences over temperature change. The second is a technical modification to simultaneously match both desired labor supply and IES values.[39] Baseline labor supply is estimated from OECD data to be $l_{2005} = 0.227$ (see Appendix C for details).

I calibrate to base year values to maintain consistency in preference parameters across model runs. That is, different fiscal scenarios may lead to different long-term labor taxes and labor supply rates. In contrast, base year labor supply and tax values are given in the data. The only exception is the first-best calibration without distortionary taxes, where I calibrate preferences to match the observed l_{2005} with $\tau_{l2005} = 0$ instead of $\tau_{l2005} = 35.19\%$. The details of the calibration are outlined in Appendix C. The benchmark calibration uses a Frisch elasticitiy of $\eta^F = 0.78$ based on a survey by Chetty, Guren, Manoli, and Weber (2011).

Finally, α_0 is chosen such that the aggregate global monetary equivalent of disutility from climate change at $2.5°C$ equals 0.37% of output as per the split in (40). The monetization of damages usually considers the world at $2.5°C$ warming in the business-as-usual (BAU) scenario, and reflects predicted global income and consumption levels at that point in time. The corresponding values are taken from a slightly modified BAU run of the 2010 DICE model (Nordhaus, 2010).[40] Labor supply at that time, which is required to compute the marginal utility of con-

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[39] Specifically, I add a preference parameter for leisure ($\phi = 1$ in standard KRB preferences). Other studies using KRB preferences usually have both the IES and the leisure preference parameter γ available as degrees of freedom to match desired moments (e.g., Jones, Manuelli, and Rossi, 1993). However, in the current study, I want to maintain consistency with the DICE model by setting $\sigma = 1.5$, thus losing one degree of freedom, which is compensated for by introducing ϕ. In Appendix C, I show that specification (41) retains consistency with a balanced growth path for the relevant ranges of the parameters. This property is important, for example, to ensure that long run growth in wages does not cause labor supply to converge toward zero.

[40] Specifically, I deactive the sea level rise module and use the slightly older damage function parameters whose calibration includes sea level rise. In addition, I modify the carbon cycle in the first period so as to reflect

sumption, is set at the baseline COMET value, since the BAU scenario represents the idea of
no tax reform. Finally, the curvature parameter α_1 is set at 2, matching the quadratic term on
output damages. Optimal carbon tax and temperature change profiles in the COMET model
without distortionary taxes for $\alpha_1 = 2$ are consistent with the modified DICE model output,
as shown in the results section 5. This finding suggests that the damage function split done in
accordance with this procedure is not driving the results of the model.

4.5   Energy Production and Emissions Abatement

There are two types of energy: carbon-based E_t^C and zero-carbon (no emissions) E_t^Z. Both
fuels are perfectly substitutable in final goods production, but zero emissions energy production
entails an additional cost over carbon-based energy. The assumption of perfect substitutability
is appropriate given the calibration of the incremental cost of clean energy as based on the cost
of emissions reductions for a given level of energy use and output from the DICE model.

The production of both types of energy requires capital and labor inputs with a Cobb-Douglas
production technology:

$$E_t = A_{Et} \cdot (K_{Et}^{1-\alpha_E} L_{Et}^{\alpha_E}) \tag{42}$$

Paired with the assumption of perfect competition in the energy production sector, formulation
(42) permits extracting the output elasticity α_E from observed expenditure shares. I use data
from the U.S. Bureau of Economic Analysis on *components of value added by industry* to calculate
labor shares. The details are provided in Appendix B. The GDP-contribution-weighted average
share of $\alpha_E = 0.403$ is used in the model.

The calibration of the costs of emissions reductions at a given energy input level (conceptually
analogous to zero emissions energy production) is based directly on the DICE model (Nordhaus,
2008). However, the costs are integrated in the model in a slightly different fashion. In the DICE
framework, a fraction of emissions *relative to the BAU scenario* (without carbon taxes) μ_t can
be eliminated at a total cost that is convex and proportional to output:

$$TC_t = \left[\phi_{1t}(\mu_t)^{\phi_2}\right] Y_t^{DICE} \tag{43}$$

where:

$$\mu_t = \frac{E_t^{c,DICE}}{E_t^{c,BAU,DICE}}$$

The parameters ϕ_{1t} and ϕ_2 are calibrated to match econometric cost function curvature esti-

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changes in base year emissions.

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mates. In addition, the marginal abatement cost at 100% emissions reductions ($\mu_t = 1$) matches estimates of an unlimited zero emissions backstop technology cost. The price of the backstop technology decreases over time, and eventually becomes cost competitive.

In the current setting, *both* carbon-based and clean energy use remain endogenous in the policy model runs. I thus translate (43) into the COMET by modeling incremental clean energy costs as:

$$TC_t = \left[\phi_{1t}(\widetilde{\mu}_t)^{\phi_2} Y_t^{DICE} \right] \equiv \Psi(E_t^{Z,COMET}) \tag{44}$$

where:

$$\widetilde{\mu}_t = \frac{E_t^{Z,COMET}}{E_t^{C,BAU,DICE}} \tag{45}$$

Formulation (44)-(45) essentially imports the DICE model's abatement cost estimates on a per-ton basis. For example, for electricity-related mitigation options (e.g., nuclear, renewables, fuel switching), the cost of producing an additional unit of wind energy in a given year is assumed to be independent of the number of coal power plants concurrently in operation. Indeed, in its review of carbon mitigation options, the IPCC tends to provide cost estimates for electricity mitigation options in terms of tons of greenhouse gas avoidance potential (IPCC Working Group III, 2007).[41] The cost per ton of clean energy above the 100% abatement level considered in the DICE model (i.e., the cost for $\widetilde{\mu}_t \geq 1$) is constant at the price of the backstop technology. To ensure that the introduction of a kink in the cost function does not pose computational problems, I alternatively represent this cost structure as separate clean energy sources, one with continuous convex cost function (44)-(45) (E_t^Z) and the other with a linear cost per ton equal to the backstop technology price p_t^{BS} in each time period ($E_t^{Z,BS}$). The energy producer's problem is thus to solve:

$$\max p_{Et}E_t - \tau_{Et}E_t^C - \Psi(E_t^Z) - p_t^{BS}E_t^{Z,BS} - w_tL_{2t} - r_tK_{2t}$$

subject to production technology (42), where $E_t = (E_t^C + E_t^Z + E_t^{Z,BS})$.

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[41]   An alternative approach would be to define emissions reductions relative to current carbon emissions [$\widetilde{\mu}_t = E_t^{Z,COMET}/E_t^{C,COMET}$], which would treat abatement cost estimates as applying on a percentage reduction basis, independent of the baseline quantity. While this approach may seem attractive to represent certain energy efficiency mitigation options, in the COMET setting, it entails the odd implication that the cost per ton of emissions reduction can be decreased by increasing carbon-based energy use. In addition, experimentation with both approaches in the COMET model suggest a much better fit for interpretation (44)-(45).

4.6 Final Consumption Good Production

Following GHKT (2011), production of the final consumption-investment good is assumed to be:

$$\widetilde{F_{1t}}(K_{1t}, L_t, E_t) = K_t^\alpha L_{1t}^{1-\alpha-v} E_t^v \qquad (46)$$

with expenditure shares $\alpha = 0.3$ and $v = 0.03$. Cobb-Douglas technology has been shown to be a poor representation of energy input use in the short-and medium run (e.g., Hassler, Krusell, and Olovsson, 2012). However, Hassler, Krusell, and Olovsson (2012) also note that the energy expenditure share in production does not appear to exhibit a clear long-run trend. Furthermore, "the possibility that the unitary-elasticity is a good approximation for the very long run cannot be excluded" (Hassler, Krusell, and Olovsson, 2012). Given the 10-year time step of the model, I follow GHKT (2011) and other studies (e.g., Leach, 2009) in working with (46) as benchmark specification.

4.7 Government

Spending

The COMET disaggregates government spending G_t into government consumption G_t^C and social transfers Ω_t (unemployment insurance, disability insurance, etc.). This distinction is in line with other calibrated Ramsey tax studies such as Jones, Manuelli, and Rossi (1997; 1993) or Lucas (1990). The consumer and government budget constraints as well as the implementability constraint need to be adjusted to incorporate transfers Ω_t. It is assumed that households take Ω_t as given. Note that transfers cannot be negative, since negative transfers (lump-sum taxes) would imply a first-best fiscal setting. As shown formally in Appendix A, the implementability constraint with social transfers is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct} C_t + U_{lt} L_t - U_{ct} \Omega_t \right] = U_{c0} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \tau_{k0}) \right\} + B_0 \right] \qquad (\text{IMP2})$$

The key government spending parameters that need to be calibrated are thus the sequence of government consumption $\{G_t^C\}_{t=0}^{\infty}$ and government transfers $\{\Omega_t\}_{t=0}^{\infty}$ to be financed. To this end, I obtain IMF Government Finance Statistics data for all available countries in the model base year (2005).[42] The PPP-adjusted GDP-weighted average share of government expenditure is 33.75% of GDP in the base year.[43] Table 4 displays the breakdown of weighted average spending

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[42] The countries covered by the IMF data account for roughly 71% of world GDP (in 2005 PPP-adjusted dollars) in 2005.

[43] One complication in combining data across countries that use cash and noncash bases of recording, respectively, is that government consumption of fixed assets is recorded in one system but not the other. Since

by category:

	% of GDP	% of Government Expenditure
Compensation of Employees	9.32	27.61
Use of Goods and Services	5.08	15.04
Grants	1.31	3.87
Subsidies	1.06	3.13
Other Expense	1.00	2.95
Social Benefits	13.32	39.45
Total	33.75	

Data sources: IMF Government Finance Statistics, IMF International Finance Statistics. Calculations exclude consumption of fixed capital for countries with noncash (accrual) basis of recording; shares recalculated given revised total expenditure estimates.

Table 4: Government Expenditure Shares (2005)

Of the expenditure items in Table 4, *Social Benefits* represents a variety of government transfers that go back to households, such as unemployment insurance, disability insurance, and most types of social security. One delicate issue is deciding the extent to which these transfers should be considered lump-sum, or independent of agents' work and consumption choices. In particular, social security benefits should be considered forced savings rather than tax-and-transfer programs to the extent that benefits depend on agents' contributions. In the literature, different authors deal with this challenge differently.

On one side of the spectrum, Prescott (2004) considers all social security payments as true tax payments, arguing that the marginal savings effect is minimal. Jones, Manuelli, and Rossi (1993) take a middle-of-the-road approach and designate 50% of social security payments as true transfers, and 50% as forced savings. On the other side of the spectrum, in the IGEM model, all social security payments are considered as forced savings (Goettle, Ho, Jorgenson, Slesnick, Wilcoxen, 2007). This choice matters because it affects the estimated distortionary cost of taxation. It should be noted that the IMF data do not include defined contribution retirement schemes or other compulsory savings schemes that "maintain the integrity of the participants' contributions" as social protection schemes (IMF, 2001). In addition, the baseline estimates of effective capital and labor income tax rates used in the calibration (discussed below) stem from studies that designate all social security tax payments as true tax payments. I thus follow Prescott's (2004) approach, and model all base year social benefits as lump-sum transfers

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government consumption of fixed assets (i) is typically a small share of GDP (1.86% on average among countries that record it), (ii) is computed using a variety of methodologies in different countries, and since (iii) I am not modeling government capital, I remove consumption of fixed assets from the data and recompute expenditure shares accordingly.

to households.

Base year government consumption and transfers are thus computed as $33.75\% - 2.68\% - 13.32\% = 17.75\%$ and 13.32% of base year GDP, respectively. Interest payments are subtracted from total government expenditure because they are accounted for separately in the model. The shares of government consumption and transfers of total government expenditure are thus 57% and 43%, respectively. The *level* of total government expenditure G_t then grows at the rates of labor productivity and population growth.[44] Government consumption in period t is then equal to $[G_t^C = G_t\,(.57)]$, and similarly government transfers are calculated as $[\Omega_t = G_t\,(.43)]$.

Lastly, the model requires estimates of baseline tax rates. Carey and Rabesona (2002) provide updated estimates of average effective tax rates across countries following the Mendoza, Razin, and Tesar (1994) methodology.[45] This procedure uses data on government revenues collected from different tax instruments to compute effective tax rates as revenues divided by the estimated size of the tax base. I calculate the base year PPP-adjusted GDP-weighted average effective tax rates for 1995-2000 based on the OECD countries in Carey and Rabesona:[46]

$$\text{Labor \& Consumption:} \quad 35.19\% \tag{47}$$
$$\text{Capital:} \quad 43.27\%$$

The benchmark calibration uses (47) as initial and, depending on the model run, as "business as usual" (no tax reform) rates.[47]

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[44] Goulder (1995) similarly models government expenditure as growing from an initial level at technology growth rate of the model, as do Jones, Manuelli, and Rossi (1993).

[45] Carey and Rabesona (2002) propose several modifications to relax assumptions made by the the Mendoza methodology. I use Carey and Rabesona's (2002) revised effective tax rate estimates, although the authors also provide updated figures using the precise Mendoza methodology through the year 2000.

[46] For capital taxes, I use Carey and Rabesona's estimates based on *net operating surplus* since those are consistent with the model's assumption that depreciation is not part of the capital tax base.

[47] The Mendoza, Razin, and Tesar methodology estimates *average* rather than marginal effective tax rates. Estimates of the latter across countries are rare. Prescott (2004) uses an adjustment factor of 1.6 to transform average non-social security labor income tax rates to marginal rates for G-7 countries. However, it is unclear to which extent this adjustment factor would apply to the rest of the world, or to capital income taxes. One would nonetheless expect this discrepancy to be a source of downward bias on estimates . Conversely, a source of upward bias is that the figures underlying (47) come exclusively from OECD member countries. However, a study by PricewaterhouseCoopers found that effective average tax rates faced by large firms incorporated in non-OECD countries (16.5%) are considerably below the non-U.S. OECD average rate (22.6%) (PWC, 2011). Applying OECD-based figures to the rest of the world may thus bias the estimates in (47) upwards. I find that using (47) as baseline values yields MCF estimates that are on the high end for capital income taxes and on the low end for labor income taxes, but within the range of the literature.

5 Quantitative Results

Computation

In order to numerically solve this infinite horizon problem, I follow a similar though slightly different approach as Jones, Manuelli, and Rossi (1993). I first optimize over all allocations for T periods as well as over the continuation gross savings rate for period T. In the benchmark calibration, $T = 25$, representing 250 years. In contrast to studies such as Jones, Manuelli, and Rossi (1993), however, one cannot impose a balanced growth path after some terminal period T in the current setting. The reason is that full effects of carbon emissions in late periods would not be accounted for due to lags in the climate system between emissions and warming. In addition, a balanced growth path requires that the climate be in steady state, that is, that carbon concentrations have stabilized. Given the assumption that clean energy backstop technologies will become fully cost competitive by the year 2255 (Nordhaus, 2010), industrial carbon emissions will stop at the latest thereafter, allowing the climate to gradually reach a new steady state.

After the last direct optimization period $T > 2255$, I thus use the continuation gross savings rate as well as the period T labor supply and period T factor distribution across sectors (i.e., the share of capital allocated to energy and final goods production) to simulate the economy and climate for another 100 years. Finally, after this additional 100 years (generally in the year 2365), I assume that the economy has reached a balanced growth path and calculate the consumption continuation value based on the theoretically calculated balanced growth path savings rate, and thus compute the present value of all future utility.[48] The optimization is performed in Matlab.

Results

Table 5 summarizes the key quantitative results for the following COMET runs:

1. An "All Taxes BAU" scenario where labor income taxes remain fixed at current levels (35.19%), there are no carbon taxes throughout the twenty-first century, and capital income taxes are varied to meet the government budget constraint.

2. An "Income Tax Reform" scenario where income taxes are optimized but there are no carbon taxes throughout the twenty-first century. This scenario measures the welfare gains from conventional tax reform as considered by the literature on optimal capital income taxes (e.g., Lucas, 1990).

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[48] In the literature, it is not uncommon to focus on a finite but very long time horizon where discounting becomes sufficiently strong such that the lack of continuation value should not affect the results noticeably (e.g., the DICE model (Nordhaus, 2008); the DSICE model (Cai et al., 2012), etc.)

3. A "BAU + Optimized Carbon Taxes + RR" scenario where labor income tax rates remain fixed at current levels (35.19%) but where carbon taxes are set optimally, and where carbon tax revenues are recycled to reduce capital income tax rates. This scenario measures the welfare gains from environmental tax reform. [Section X below explores the impacts of assuming alternative revenue recylicng scenarios.]

4. A "BAU + 'Wrong' Carbon Taxes + RR" scenario which is identical to (3) except that carbon taxes are set at first-best levels that would be optimal if there were no distortionary taxes. These levels correspond to the Pigouvian tax or the social cost of carbon in a setting without distortionary fiscal policy. The difference in welfare between (4) and (3) reflects the additional value of the consideration of distortionary tax interactions in the design of carbon taxes.

5. An "Income Tax Reform + Optimized Carbon Taxes" scenario that represents full optimization over all tax instruments in a second-best setting.

6. An "Income Tax Reform + 'Wrong' Carbon Taxes" scenario where income taxes are optimized but carbon taxes are set at first-best levels as in (4). The difference between (5) and (6) once again reflects the additional value of the consideration of distortionary tax interactions in the design of carbon taxes.

7. A "First-Best" scenario where the government is allowed to raise revenues by imposing non-distortionary lump-sum taxes, and optimally levies first-best carbon taxes. Once again, these taxes correspond to the Pigouvian rate or the social cost of carbon evaluated at the optimal allocation. This scenario represents the common implicit assumption in the integrated assessment model literature.

Table 5 summarizes the quantitative results of the main fiscal scenarios considered:

Fiscal Scenario		Capital Tax	Labor Tax	Carbon Tax		MCF	T_t	ΔWelfare[1]	
		Avg.	Avg.	\$/mtC		Avg.	$C°$	\$2005 bil.	%ΔC_t
Income Taxes:	Carbon Tax:	2025-2255	2025-2255	2015	2025	2025-2255	Max	ΔC_{2015}	$\forall t$
BAU (τ_l fixed)	None (until 2115)	35%	35.19%	0	0	1.52	4.57	0	0%
Optimized	None (until 2115)	2.3%[3]	40%	0	0	1.05	4.55	\$24,451	0.83%
BAU (τ_l fixed) + RR[2]	Optimized	31.4%	35.19%	\$43[4]	\$65[4]	1.44	3.34	\$24,971	0.85%
BAU (τ_l fixed) + RR[2]	'Wrong' (first-best)	32%	35.19%	\$67	\$99	1.45	2.93	\$23,440	0.80%
Optimized	Optimized	2.4%[3]	40%	\$55	\$81	1.05	3.25	\$45,237	1.50%
Optimized	'Wrong' (first-best)	2.3%[3]	40%	\$67	\$99	1.05	2.95	\$44,642	1.49%
First-Best (lump sum)	Optimized	0	0	\$67	\$99	1.00	2.96	[\$83,285][5]	[2.66%][5]

[1]Relative to all tax BAU scenario (without carbon taxes). Measured as equivalent variation change in aggregate initial consumption ΔC_{2015} or as permanent %ΔC_t through 2265.

[2]Carbon tax revenues used to reduce capital income tax rates ("revenue recycling," RR).

[3]Consists of high initial tax (46% in 2025) followed by $\sim$ 0% tax (except for pre-simulation period $T = 2265$; see Section 5.)

[4]Defined as the difference between total excise taxes on carbon-based and clean energy, respectively. The government taxes all types of energy use in this setting because it increases the marginal product of labor and tightens the constraint that labor taxes can only be 35%.

[5]Calculation uses utility function parameters from second-best model runs to evaluate and compare *both* first- and second-best allocations. In reality, leisure preferences are calibrated differently in the first-best setting because ($\tau_{I0} = 0$), whereas ($\tau_{I0} = 35\%$) in all other model runs. Leisure preferences needed to rationalize labor supply thus differ across the scenarios, making welfare calculations not strictly comparable.

Table 5: Main Results

Three main insights emerge from the results in Table 5

First, optimal carbon levies are consistently lower when there are distortionary taxes. Figure 5 displays optimal carbon tax schedules from the key model runs (3), (5), and (7), as well as optimal carbon taxes from a slightly modified 2010 DICE model run (see footnote 39) for comparison:

Optimal Carbon Tax Paths
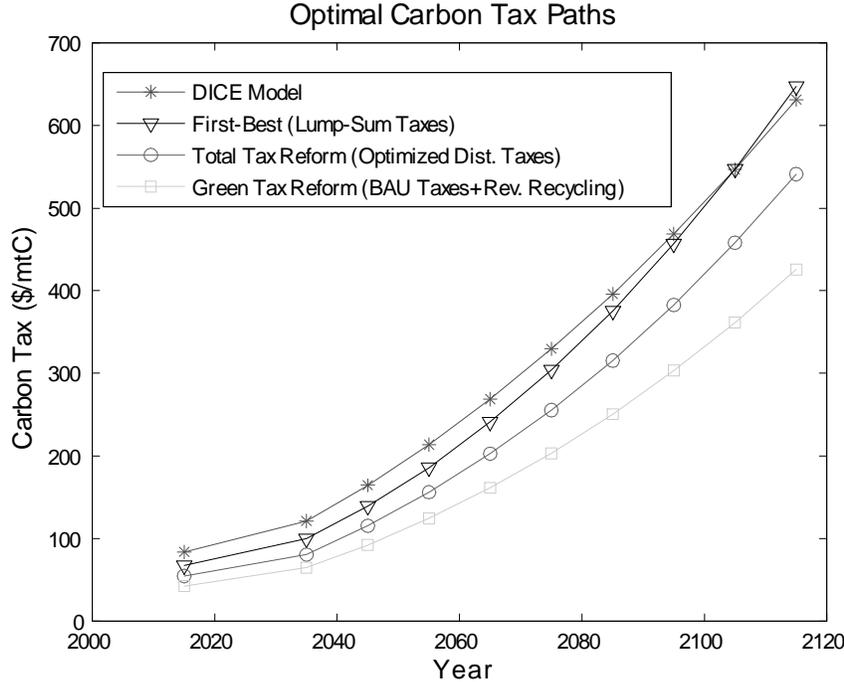
Throughout the century, the optimal carbon taxes are 20% to 35% lower when levied alongside distortionary taxes. They start at $55/mtC$ ($43/mtC$) in 2015, rising to $541/mtC$ ($426/mtC$) by 2105 in the scenario with (without) income tax reform. The change in optimal carbon taxes in a setting with distortionary fiscal policy is driven by two factors. First, the size of the economy is smaller. As a result, the value of marginal damages is lower. For example, on average over the time horizon 2015 to 2255, output in the COMET income tax BAU run (scenario (3)) is 8.9% lower than in the DICE model. Another implication of a smaller economy is that the level of carbon taxes needed to achieve a given temperature change target is lower. Second, in a setting with distortionary taxes where the marginal cost of public funds exceeds one, optimal carbon taxes are set below the value of marginal damages (the Pigouvian tax), as discussed in the theory results.

These quantitative findings compare optimal carbon taxes across settings with and without distortionary taxes. In contrast, the theoretical analysis focused on optimal carbon taxes compared with Pigouvian taxes entirely within a setting with distortionary fiscal policy (that is, evaluated at the second-best allocation). The reason for this slight shift in comparison is that

the integrated assessment model literature estimates first-best carbon tax schedules. Pigouvian taxes evaluated at the optimal allocation without distortionary taxes are thus the appropriate comparison group to represent the literature.

As emphasized by Metcalf (2003), it is essential to evaluate both how distortionary taxes affect optimal pollution prices and the associated changes in optimal quantities. In particular, he finds that environmental quality may be higher in a world with distortionary taxes and lower economic activity, despite lower pollution tax rates. Figure 5 illustrates optimal temperature change in model runs (3), (5), (7), and the DICE comparison run:
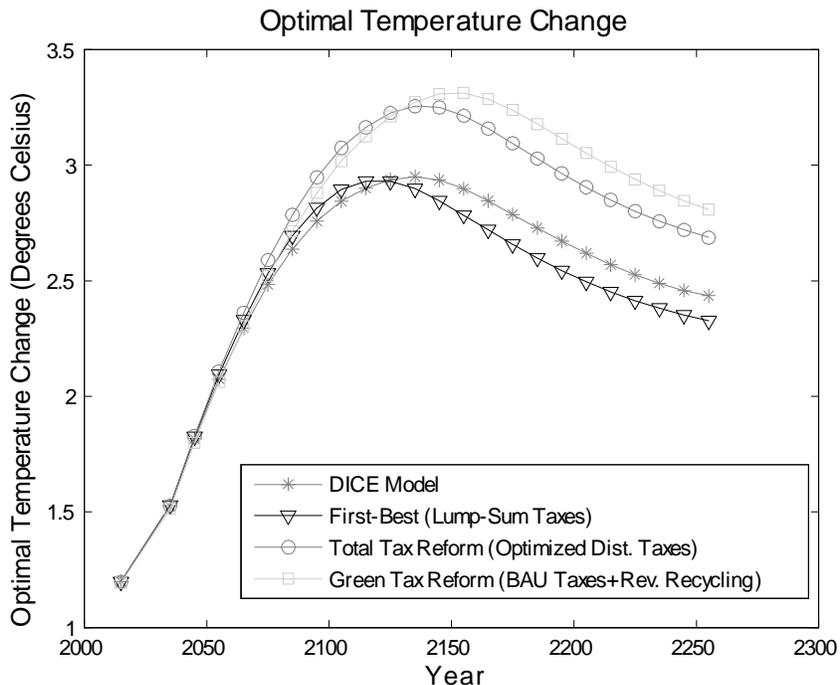
Optimal peak temperature change (in °C) is projected to be between 11% and 13% higher when there are distortionary taxes. In line with Metcalf's (2003) results, the difference in temperature change between the environment with BAU income taxation and with optimized income taxation is thus quite small, despite the fact that carbon tax rates are much lower in the former scenario. The desirability of higher temperature change is essentially a reflection of the increased social marginal emissions reduction costs in the setting with distortionary taxes.

The second main result is that consideration of fiscal policy in the design of carbon taxes produces large net welfare gains. Specifically, I compare the welfare gains from imposing optimized carbon taxes (model runs (3) and (5)) to the welfare gains from imposing carbon taxes that were designed for a setting without distortionary taxes (model runs (4) and (6)). The additional welfare gain from setting adjusted carbon taxes is $1.5 trillion ($2005 lump-sum consumption

equivalent) in the BAU income tax scenario, and $595 billion in the optimized income tax scenario. While this welfare gain is modest as a percentage of the total welfare gain from carbon taxes, in levels it is arguably quite large.

Finally, the welfare costs of failure to enact carbon taxes appear to be on the same order of magnitude as the welfare costs arising from capital income taxation. Estimating the latter cost in a global aggregate model with a single type of physical capital is, of course, a gross approximation. However, it should be noted that the estimated 0.83% consumption equivalent welfare gain from the optimal capital income tax phase out is very much in line with Lucas' (1990) estimates of the likely range for the U.S. economy (between 0.75% and 1.25%). This result suggests that positive capital income taxes and the *absence* of carbon taxes are not only qualitatively analogous as suggested by the theoretical results, but that the respective welfare losses from each policy are quantitatively of the same magnitude as well.

To summarize, there are three main quantitative results. First, the optimal carbon tax schedule is $20-35\%$ lower when there are distortionary taxes. However, optimal peak temperature change is only between 11% and 13% higher in an environment with distortionary taxes. Second, the welfare gains from adjusting carbon taxes to account for their fiscal impacts is between $595 billion and $1.51 trillion, depending on the tax reform scenario. Third, the welfare gains from income tax reform that optimally phases out capital income taxes is of similar size as the welfare gains from an environmental tax reform which imposes optimal carbon levies and uses their revenue to reduce, but not eliminate, capital income tax rates.

6 Extensions

This section formally considers three extensions of the core model. First, I theoretically explore two cases with positive capital income taxes. Second, I extend both the theoretical and the quantitative model to incorporate non-renewable energy resource dynamics. Finally, I discuss the implications of non-separability in preferences over the climate, consumption, and leisure.

6.1 Positive Capital Income Taxes

Upper Bound on Capital Income Tax Rates

Following the treatment by Atkeson, Chari, and Kehoe (1999), consider adding an upper bound on the capital tax rate that the government can set. The rationale behind this assumption is as follows. If the government imposes capital taxes that are too high, consumers can always choose not to rent their capital out to firms and to earn a return of $(1 - \delta)K_t$ instead. This return provides a lower bound on the equilibrium return to capital, which, in turn, defines the upper

bound on capital taxes that can be supported in competitive equilibrium:

$$1 - \delta \leq \{1 + (r_{t+1} - \delta)(1 - \tau_{kt+1})\} = \frac{U_{ct}}{\beta U_{ct+1}} \tag{48}$$

Revisiting the planner's problem with this additional constraint leads to the following proposition:

Proposition 4 *If consumer preferences are of the form:*

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} + \vartheta(L_t) + v(T_t) \tag{49}$$

or

$$U(C_t, L_t) = \frac{(C_t L_t^{-\gamma})^{1-\sigma}}{1 - \sigma} + v(T_t) \tag{50}$$

(with $\gamma > 0$), if capital is necessary in final goods production $(F_{1t}(0, L_t, E_t) = 0)$, and if the capital tax rate is bounded above by the agent's ability to hold capital without renting it out to firms (48), then:

(i) Optimal capital taxes are positive and at the upper bound for a finite number of periods, intermediate for one period, and then drop to zero forever.

(ii) Optimal carbon taxes on output damages are less than Pigouvian while carbon taxes are positive, and jump to Pigouvian levels two periods after the capital tax upper bound ceases to bind.

Proof: See Appendix A.

The intuition for this result is straightforward. As long as the government imposes maximal capital income taxes, it distorts households' savings decisions. That is, the planner creates a wedge between the marginal rates of substitution and transformation for present and future consumption. This wedge likewise implies the optimality of a less-than-Pigouvian carbon tax, as per the intuition discussed in Section 3.

Exogenously Given Capital Income Tax Rate

Suppose now that there is an exogenously given constraint that the capital income tax rate be equal to some level $\overline{\tau_k} \in (0, 1)$. Such an assumption may reflect unmodeled political constraints on the government's ability to enact optimal tax policies. From the consumer and final goods producer's first order conditions for capital, this constraint can be formalized as:

$$\frac{U_{ct}}{\beta U_{ct+1}} = 1 + (1 - \overline{\tau_k})(F_{kt+1} - \delta) \tag{51}$$

for all $t > 0$. In this setting, the impacts of changes in energy use, factor allocation to energy production, and temperature change on the tightness with which (51) binds all figure into the optimal carbon tax formulation. (See Appendix A for the derivation and details.)

Importantly, *production damages* from climate change now enter the optimal carbon tax formulation in two ways. On the one hand, they decrease welfare directly by reducing available resources in future periods as shown in the benchmark expression (24). These future output losses are now discounted at a higher rate than households' intertemporal marginal rate of substitution due to the intertemporal wedge in (51). If this were the only difference to the benchmark model with production damages, the optimal carbon tax would thus be less-than-Pigouvian.

However, output losses also interact with the capital tax constraint. Consider the case where the optimal capital tax is below $\overline{\tau_k}$. The net-of-tax marginal rate of transformation faced by agents when making their savings decisions is thus lower than the planner would have wanted it to be. Climate change production losses decrease the marginal product of capital in future periods even further away from the unconstrained optimum. That is, climate change exacerbates the capital income tax constraint in (51). This interaction provides the planner with an additional incentive to avoid climate change. Ceteris paribus, this effect thus *increases* the optimal carbon tax to internalize output damages. In sum, the exogenous capital income tax can in principle increase or decrease the optimal charge on production damages from climate change relative to the Pigouvian rate.

There are additional variables related to carbon taxes that interact with constraint (51) and alter the optimal total carbon tax formulation. For example, decreased energy use may decrease the marginal product of capital as well, depending on the complementarity between capital and energy in production. The optimal *total* carbon tax is thus also ex-ante ambiguously affected by the capital income tax constraint (51)). See Appendix A for a further discussion.

6.2   Nonrenewable Resources

Assume now that carbon energy is in finite supply with initial stock R_0 in the ground. To focus on the central mechanisms, further assume that this carbon resource can be extracted costlessly, and that there is no alternative energy source. With a competitive fossil fuel production sector, the representative firm maximizes the present value of profits subject to its fuel resource constraint:

$$\max \sum_{t=0}^{\infty} q_t(1 - \tau_{\pi t})\{(p_{Et} - \tau_{Et})E_t\}$$

$$+ \sum_{t=0}^{\infty} q_t \widetilde{\mu}_t \left[R_t - E_t - R_{t+1}\right]$$

Here, q_t denotes the relative price of consumption in period t (expressed in period 0 units). R_t and R_{t+1} represent the stock of the fossil fuel left in the ground at the beginning of periods t and $t+1$, respectively. Hotelling profits taxes at time t are denoted by $\tau_{\pi t}$. The firm's first order conditions with respect to extraction E_t and the remaining fossil fuel stock R_{t+1} are, respectively:

$$(1 - \tau_{\pi t})(p_{Et} - \tau_{Et}) = \widetilde{\mu}_t \tag{52}$$

$$q_t \widetilde{\mu}_t = q_{t+1} \widetilde{\mu_{t+1}} \tag{53}$$

Combining equations (52) and (53) yields the standard Hotelling condition that the after-tax price of carbon energy rises at the rate of interest:

$$(1 - \tau_{\pi t})(p_{Et} - \tau_{Et}) = \frac{q_{t+1}}{q_t}(1 - \tau_{\pi t+1})\left(p_{Et+1} - \tau_{Et+1}\right) \tag{HOT}$$

Expression (HOT) demonstrates the well-known result that a constant Hotelling profit tax rate on non-renewable resource producers does not affect extraction behavior (see, e.g., Dasgupta and Heal, 1979). The relative returns to oil production across time periods determine optimal extraction schedules. As a result, decreasing fossil fuel profits in each period equiproportionally does not affect producers' incentives. In other words, constant Hotelling profit tax rates on oil production are non-distortionary. If possible, the government thus optimally sets these taxes equal to 100%.

On the other hand, if Hotelling profit taxes are not available, nonrenewable resource rents remain in the agent's budget constraint and hence in the implementability constraint. In order to employ the primal approach to characterizing optimal taxes, these profits must be expressed strictly in terms of allocations. In addition, one needs to prove that the optimal allocation can be decentralized by appropriately designed prices and policy instruments. In Appendix A, I formally show that this can be done *for a given initial carbon tax* $\overline{\tau_{E0}}$, and that the implementability constraint in this setting becomes:

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}C_t + U_{lt}L_t\right] = U_{c0}\left[K_0\left\{1 + (F_{k0} - \delta)(1 - \overline{\tau_{k_0}})\right\} + B_0 + \sum_{t=0}^{\infty} E_t\left(F_{E0} - \overline{\tau_{E0}}\right)\right] \tag{54}$$

Note that the initial emissions tax $\overline{\tau_{E0}}$ and the carbon resource endowment R_0 both need to be added as initial conditions to the definitions of competitive equilibrium and Ramsey equilibrium in this setting. If profits can be fully taxed, the form of the implementability constraint is as in (IMP).

To facilitate analytic inference on the structure of optimal carbon taxes in this setting, I assume that preferences are of the commonly used forms (49) or (50).

Proposition 5 *Assume preferences are of the form (49) or (50).*

If profit taxes are not available, the optimal carbon tax at time $t > 0$ is implicitly defined by:

$$\tau_{Et}^* = \left(\frac{\tau_{Et}^{Pigou,U}}{MCF_t} \right) + \tau_{Et}^{Pigou,Y} + \kappa \left[1 - \frac{1}{MCF_t} \right] (\widetilde{\mu}_t) \tag{55}$$

where κ is a constant equal to $(1-\sigma)^{-1}$ for (49) and $\kappa = ((1-\sigma)(1-\gamma))^{-1}$ for (50).

If 100% profits taxes are available, the optimal carbon tax at time $t > 0$ is implicitly defined by:

$$\tau_{Et}^* = \left(\frac{\tau_{Et}^{Pigou,U}}{MCF_t} \right) + \tau_{Et}^{Pigou,Y} \tag{56}$$

Proof: See Appendix A.

To summarize, Proposition 6 reveals that the optimal carbon tax in a setting with nonrenewable energy sources and distortionary taxes is structured similarly as in the core model setting with constant returns to scale in energy production. However, a difference arises if the government cannot tax away oil producers' Hotelling profits. In that case, optimal carbon taxes are increased as a means of indirectly capturing fossil fuel producers' Hotelling rents.[49] The finding of higher optimal pollution taxes when they can serve as proxy for taxes on rent/profits is not new (see, e.g., Bento and Jacobsen, 2007; Fullerton and Kim, 2008; Williams, 2002).

Appendix C provides the details and results of a quantitative implementation of the COMET model with non-renewable energy inputs. Specifically, this extension replaces the clean and carbon-based energy production sector of the benchmark model with the fossil fuel energy production sector of Golosov, Hassler, Krusell, and Tsyvinski (2011). This specification features a non-renewable energy resource with comparatively high energy content per ton of carbon emissions and zero extraction costs ("oil"), a carbon-based energy form producible from labor inputs ("coal"), and a backstop technology which becomes available in 2120. The optimal energy production trajectory thus begins with an oil-only regime, which lasts until economically viable petroleum reserves are exhausted.[50] A coal regime follows, until alternative energy forms become available in 2120. The key quantitative findings are as follows. First, optimal peak temperature change (in $°C$) is between 1% and 12% higher when there are distortionary taxes. The absolute

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[49] It may seem surprising that the planner finds it optimal not to tax capital income after the first period *even when there are untaxable profits*. The literature has often shown the existence of untaxable profits to imply the optimality of taxing capital income even with preferences such as (49) or (50) (e.g., Jones, Manuelli, and Rossi, 1997). The reason for this difference is that that profits in the current setting - the Hotelling rents from resource extraction - do not depend on the capital stock after period zero.

[50] In alternative calibrations, it may not be desirable to use up all petroleum (see Golosov, Hassler, Krusell, and Tsyvinski, 2011, for a discussion). However, within the context of my model, I find that oil is exhausted in all scenarios considered.

level of optimal temperature change is considerably higher than in the benchmark model even without distortionary taxes because of the difference in clean energy availability. Second, optimal carbon taxes during the oil regime may be considerably higher when there are distortionary taxes, particularly if profit taxes are not available. However, the precise level of the optimal tax during the oil regime is uniquely determined only for a given initial period carbon tax assumed to be in place from $2015 - 2025$. Third, optimal carbon taxes during the coal regime are between 8% and 12% lower when there are optimized distortionary taxes, similar to the benchmark model results. For further discussions of climate policy across energy regimes, see, e.g., Golosov, Hassler, Krusell, and Tsyvinski (2011), or van der Ploeg and Withagen (2012).

6.3   Non-Separable Environmental Preferences

When preferences are non-separable in consumption, leisure, and temperature change, the planner's problem is still given by (16), with the key difference that U_{ct} and U_{lt} are now functions of temperature change T_t. As a result, the marginal damage of temperature change in period $t > 0$ (in utils) is now given by:

$$\underbrace{U_{Tt}}_{\text{Utility damage}} + \underbrace{\lambda_{1t}\frac{\partial Y_t}{\partial T_t}}_{\text{Output damage}} + \underbrace{\phi[U_{cTt}C_t + U_{lTt}L_t]}_{\text{Offer curve impact}} = \underbrace{-\xi_t}_{\text{Marginal damage from } T_t} \tag{57}$$

where ϕ is the Langrange multiplier on the competitive equilibrium implementability constraint. In addition to utility losses and output damages, climate change can thus impact welfare in a third way in this setting. If temperature affects households' offer curves, it changes the set of allocations that can be decentralized as a competitive equilibrium. In other words, the government may not be able to induce households to supply the same amount of labor or choose the same consumption paths if temperature change affects households' marginal utilities of consumption and leisure. For example, if climate change is complementary with leisure, households will be less willing to supply labor at a given wage as the climate warms.

Combining the planner's first order conditions for energy inputs E_t, total labor supply L_t, and energy sector labor L_{2t} with (57), and comparing with the energy producer's optimality conditions (12) leads to the following expression implicitly defining the optimal tax in this setting:

$$\tau_{Et}^{*} = \sum_{j=0}^{\infty} \beta^{j} \left\{ \frac{U_{Tt+j}}{\lambda_{1t}} + \frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial Y_{t+j}}{\partial T_{t+j}} + \frac{\phi}{\lambda_{1t}} [U_{cTt+j} C_{t+j} + U_{lTt+j} L_{t+j}] \right\} \frac{dT_{t+j}}{dE_t} \qquad (58)$$

$$= \underbrace{\frac{\tau_{Et}^{Pigou,U}}{MCF_t}}_{\text{Utility damages}} + \sum_{j=0}^{\infty} \beta^{j} \left\{ \underbrace{\frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial Y_{t+j}}{\partial T_{t+j}}}_{\text{Output damages}} + \frac{(MCF_t - 1)}{MCF_t} \underbrace{\left[\frac{U_{cTt+j} C_{t+j} + U_{lTt+j} L_{t+j}}{U_{cct} C_t + U_{ct} + U_{lct} L_t} \right]}_{\text{Offer curve impacts}} \right\} \frac{dT_{t+j}}{dE_t}$$

where the second equation follows from substituting in for ϕ from the planner's first order condition for C_t, multiplication by U_{ct}/U_{ct}, and invoking the definition of the marginal cost of public funds (17).

Expression (58) leads to two main insights. On the one hand, non-separability in preferences does not change the key theoretical findings of this paper. That is, it is still the case that (i) utility damages are internalized differently from production damages, and (ii) the optimal tax component for output damages is Pigouvian if the planner optimally sets capital income taxes to zero from time t onwards (implying that $\frac{\lambda_{1t+j}}{\lambda_{1t}} = \frac{U_{ct+j}}{U_{ct}}$ for all $j \geq 0$).

On the other hand, the optimal total carbon tax can now be larger or smaller than the Pigouvian tax, depending on the sign and size of the size and sign of the impacts of temperature change on the agents' offer curves. In particular, the offer curve impact will increase the level of the optimal carbon tax if climate change utility impacts are *complementary with leisure* and *a substitute for consumption*. Intuitively, this result goes back to the Corlett and Hague (1953) rule that goods which are relative complements to leisure should be taxed relatively more. Relatedly, Schwarz and Repetto (2000) demonstrate that the welfare costs of the interaction between labor income taxes and pollution taxes are reduced to the extent that improved environmental quality can increase labor supply. Carbone and Smith (2008) provide a quantitative analysis of the implications of non-separability in a model of particulate matter pollution in the U.S. economy.

Unfortunately, the literature provides very little empirical evidence on the likely magnitudes and signs of the complementarity between climate change and leisure and consumption, respectively. The climate amenity value estimates underlying the DICE damage function include modest increases in the value of time use for cold regions and negative impacts for warm regions, based on moderate but positive time use value estimates for the United States by Nordhaus (1998). Neidell and Zivin (2010) find that overall labor supply in the United States does not appear responsive to changes in weather-induced temperature variation. However, labor supply in climate-sensitive industries (agriculture, construction, utilities, etc.) does decrease significantly and sizably during hot weather. A well-known concern in extrapolating from weather variation impacts to climate change is that they do not account for long-term adaptation (e.g., Mendel-

sohn, Nordhaus, and Shaw 1994). Indeed, Neidell and Zivin do find that the impacts of warm temperatures are weaker in warmer regions. Future research in this area would thus be highly valuable for more accurate calibration of environmental tax interaction studies.

7   Conclusion

This paper considers the optimal taxation of carbon jointly with distortionary taxes enacted to raise government revenues. Specifically, I theoretically characterize and then quantify optimal dynamic carbon taxes as a part of fiscal policy in a climate-economy model based on the world economy. The three main results of the paper can be summarized as follows.

First, I demonstrate both a theoretical and quantitative link between capital and carbon taxes. On the theoretical side, I formally show that the optimal carbon tax to internalize production losses from climate change is the Pigouvian tax whenever capital income taxes are optimally set to zero. Intuitively, this is because setting carbon taxes below Pigouvian rates distorts incentives to invest in the environmental capital stock of the climate. This is analogous to capital income taxes, which distort incentives to invest in physical capital. On the quantitative side, I estimate that the welfare costs of continuing our current policy of *not* taxing carbon are of similar magnitude as the welfare costs of taxing capital income ($25 trillion, $2005 lump-sum consumption equivalent; 0.84% permanent consumption increase).

Second, I theoretically motivate and quantify a distinction between production and direct utility impacts of climate change. On the theoretical side, the intuition for this result is that utility damages reflect the value of the climate as final consumption good. Conversely, production damages reflect the value of the climate as intermediate input to production. The optimal carbon tax internalizes these damages differently. Based on the seminal climate change impact estimates from the DICE model (Nordhaus, 2008), and a new damage function component to capture long-term labor productivity impacts from malaria exposure, I estimate that 70% of climate change impacts from 2.5° affect production; 30% affect utility directly.

Third, I quantify optimal carbon tax schedules across several fiscal scenarios. Compared to the setting with lump-sum taxation considered by the literature, I find that the optimal carbon price path is 20% lower when there are optimized distortionary taxes, and 35% lower when there are business-as-usual distortionary taxes. I estimate that adjusting carbon taxes to take into account distortionary tax interactions increases the welfare gains from climate policy by $595 billion to $1.5 trillion.

I would like to conclude by discussing three potential extensions of this study.

First, this paper estimates optimal carbon taxes from a global planner's perspective in a globally aggregated economy. This is the natural starting point for an analysis of optimal climate

policy, which depends on the aggregate global emissions reduction costs and benefits. However, an equally natural next step is to consider a disaggregated model with heterogeneous regions. An important difficulty in the multi-region setting is accounting for (potentially strategic) interactions and spillovers across regions, such as through trade or fossil fuel markets (see, e.g., Hassler and Krusell, 2012). Regionally differentiated models have been considered in the integrated assessment literature (e.g., the RICE model, Nordhaus, 2010), by theoretical studies on optimal carbon taxes in dynamic competitive equilibrium economies (Hassler and Krusell, 2012; Krusell and Smith, 2012), and in positive empirical work on carbon tax interactions with other taxes (e.g., Babiker, Metcalf, and Reilley, 2003; Bernard and Vielle, 2003). A multi-region version of COMET would thus build on the central elements of these studies.

Second, this paper focuses on a deterministic setting, again as a natural benchmark. Climate-economy models have considered uncertainty in a variety of forms (parametric uncertainty, stochasticity, autonomous learning, endogenous learning, etc., see, e.g., Peterson, 2006). For example, Lemoine and Traeger (2012) find that consideration of uncertainty over tipping points or irreversibilities in the climate system can increase optimal carbon levies compared to a benchmark based on the DICE model. It is unclear how consideration of such tipping points would interact with distortionary taxes. Several recent models also consider both climate and economic uncertainty (e.g., Krusell and Smith, 2012; Cai, Judd, and Lontzek, 2012). Briggs (2012) incorporates uncertainty over abatement costs in a dynamic setting with (abatement) capital accumulation. Climate policy and business cycles have further been considered by studies such as Heutel (2012) and Fischer and Springborn (2011). A stochastic version of the COMET could consider uncertainty in yet another direction: fiscal fluctuations. Chari and Kehoe (1998) find that optimal labor, capital, and asset taxes vary differentially in response to fiscal shocks. It would correspondingly be interesting to study the optimal response of carbon taxes to fiscal shocks, particularly in light of this paper's finding that optimal capital and carbon taxes are closely linked.

Third, consideration of endogenous technical change in climate-economy models can alter optimal policy prescriptions (see, e.g., Popp, 2004, Acemoglu, Aghion, Bursztyn, and Hemous, 2012). For example, Acemoglu, Aghion, Bursztyn, and Hemous (2012) propose a combination of carbon taxes and clean energy research subsidies. However, their analysis allows for lump-sum taxation to finance subsidies. In the context of an endogenous growth model with environmental degradation, Fullerton and Kim (2008) argue that pollution tax revenue may generally be insufficient to finance optimal levels of public abatement research spending. It would thus be interesting to reconsider the optimal policy mix between research subsidies and carbon taxes in a calibrated climate-economy model where subsidies have to be financed through distortionary taxation.

For many countries around the world, the current fiscal outlook is gloomy (see, e.g., Reinhart and Rogoff, 2011). This study has argued that carbon taxes have to be designed with care to account for their potentially adverse effects on other tax bases, such as employment. However, this study also found that the imposition of appropriately designed carbon taxes would yield substantial benefits, both in terms of raising revenues and by improving intertemporal production efficiency.

A Appendix A: Proofs

A.1 Proposition 1

Consider first the representative household's first order conditions. Note that I assume throughout that the solution to the household's problem is interior.

Letting γ_t be Lagrange multiplier on the consumer's flow budget constraint (3) in period t, his first order conditions are given by:

$[C_t]$:

$$\gamma_t = \beta^t U_{ct} \tag{A.1}$$

$[L_t]$:

$$\frac{-U_{lt}}{U_{ct}} = w_t(1 - \tau_{lt}) \tag{A.2}$$

$[K_{t+1}]$:

$$\gamma_t = \beta\gamma_{t+1}\left\{1 + (r_{t+1} - \delta)(1 - \tau_{kt+1})\right\} \tag{A.3}$$

$[B_{t+1}]$:

$$U_{ct}\rho_t = \beta U_{ct+1} \tag{A.4}$$

Next, consider the final goods producer's problem, which is to choose L_{1t}, K_{1t}, and E_t to solve:

$$\max F(T_t, K_{1t}, L_{1t}, E_t) - w_t L_t - p_{Et}E_t - r_t K_t$$

Letting F_{jt} denote the first derivative of the production function with respect to factor j, the associated first order conditions are:

$$
\begin{aligned}
F_{lt} &= w_t \\
F_{Et} &= p_{Et} \\
F_{kt} &= r_t
\end{aligned}
\tag{A.5}
$$

The energy producer solves:

$$\max(p_{Et} - \tau_{Et})E_t - w_t L_{2t} - r_t K_{2t}$$

subject to:

$$E_t = F_{2t}(L_{2t}, K_{2t})$$

The associated FOCs are:

$$
\begin{aligned}
(p_{Et} - \tau_{Et})F_{2lt} &= w_t \\
(p_{Et} - \tau_{Et})F_{2kt} &= r_t
\end{aligned}
\tag{A.6}
$$

A.2 Proof of Proposition 1

Note: Since the quantitative model incorporates both government consumption G_t as well as non-negative social transfers Ω_t which are provided to households (e.g., unemployment insurance, disability insurance, etc.), this proof incorporates both types of government spending. The only difference from the core model as set up in Section 2 is that Ω_t has to be added to the consumer budget constraint (3)and subtracted from the government budget constraint (13) in each period.

Direction: If the allocations and initial conditions constitute a competitive equilibrium, then constraints (RC)-(IMP) are satisfied. If we are in a competitive equilibrium, the consumer's FOCs (A.1)-(A.4) will be satisfied. Note that we can multiply both sides on the FOC for capital savings (A.3) by K_t to find that:

$$\left[\gamma_t - \gamma_{t+1}\left\{1 + (r_{t+1} - \delta)(1 - \tau_{kt+1})\right\}\right]K_{t+1} = 0 \tag{A.7}$$

Similarly, for bonds we have that:

$$\left[\gamma_t \rho_t - \gamma_{t+1}\right]B_{t+1} = 0 \tag{A.8}$$

Also note that the consumer's transversality conditions necessarily hold in a competitive equilibrium:

$$\lim_{t \to \infty} \gamma_t B_{t+1} = 0 \tag{A.9}$$
$$\lim_{t \to \infty} \gamma_t K_{t+1} = 0$$

In a competitive equilibrium, the consumer's flow budget constraint (3) also needs to be satisfied. Multiplying both sides of the flow budget constraint in each period by γ_t yields:

$$\gamma_t\left[C_t + \rho_t B_{t+1} + K_{t+1}\right] = \gamma_t\left[w_t(1 - \tau_{lt})L_t + \left\{1 + (r_t - \delta)(1 - \tau_{kt})\right\}K_t + B_t + \Omega_t + \Pi_t\right] \tag{A.10}$$

Note that energy sector profits in competitive equilibrium will be equal to zero,[51] given the assumptions of constant returns to scale and perfect competition in energy production. Taking note of this fact and summing equation (A.10) over all t leads to:

$$\sum_{t=0}^{\infty} \gamma_t\left[C_t + \rho_t B_{t+1} + K_{t+1} - w_t(1 - \tau_{lt})L_t - \left\{1 + (r_t - \delta)(1 - \tau_{kt})\right\}K_t - B_t - \Omega_t\right] = 0 \tag{A.11}$$

Except for the time zero bonds and capital return, all of the other terms relating to capital and bond cancel out of equation (A.11) out as per equations (A.7), (A.8) and the transversality

---

[51] To see this formally, substitute the energy producer's FOCs into the definition of energy sector profits:

$$\Pi_t = (p_{Et} - \tau_{Et})F(K_{2t}, L_{2t}) - F_{l2t}(p_{Et} - \tau_{Et})L_{2t} - F_{kt}(p_{Et} - \tau_{Et})K_{2t}$$

If $F(K_{2t}, L_{2t})$ exhibits constant returns to scale, by Euler's theorem for homogenous functions, $(F(K_{2t}, L_{2t}) = F_{l2t}L_{2t} + F_{k2t}K_{2t})$, and this expression reduces to zero.

conditions (A.9). We thus end up with:

$$\sum_{t=0}^{\infty} \gamma_t \left[C_t - w_t(1 - \tau_{lt})L_t - \Omega_t \right] = \gamma_0 \left[K_0 \left\{ 1 + (r_0 - \delta)(1 - \tau_{k_0}) \right\} + B_0 \right] \qquad \text{(A.12)}$$

Next, based on the consumer's and firm's FOCs, one can substitute out for γ_t, $w_t(1 - \tau_{lt})$, and r_0 in (A.12) to obtain the implementability constraint (IMP):

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}C_t + U_{lt}L_t - U_{ct}\Omega_t \right] = U_{c0} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \tau_{k_0}) \right\} + B_0 \right] \qquad \text{(A.13)}$$

We have thus shown that competitive equilibrium implies that the implementability constraint is satisfied.

Next, to show that the final goods resource constraint (RC) holds in competitive equilibrium, (i) add up the consumer and government flow budget constraints (3) and (13), (ii) substitute in the bond market clearing condition (14), (iii) invoke the definition of energy sector profits (10), (iv) substitute in capital and labor market clearing conditions (11), (v) substitute in for factor prices based on the final good producer's FOCs (A.5), and (vi) invoke Euler's theorem based on the assumption of constant returns to scale in final goods production.

Finally, the carbon cycle constraint (CCC) and the energy producer's resource constraint (ERC) hold by definition in competitive equilibrium.

Direction: If constraints(RC)-(IMP) are satisfied, one can construct competitive equilibrium. This direction of the proof proceeds by construction. First, let factor prices be given by:

$$\begin{aligned} F_{lt} &= w_t \qquad\qquad\qquad\qquad\qquad\qquad \text{(A.14)} \\ F_{Et} &= p_{Et} \\ F_{kt} &= r_t \end{aligned}$$

These factor prices are obviously consistent with profit maximization in the final goods sector, as required in a competitive equilibrium. Next, let the return on bonds be given by:

$$\rho_t = \beta U_{ct+1}/U_{ct}$$

Again, this price is clearly consistent with utility maximization as per the agent's FOC (A.4). One could decentralize an

Let the labor tax rate be determined by:

$$\begin{aligned} -U_{lt}/U_{ct} &= (1 - \tau_{lt})F_{lt} \\ 1 + \frac{U_{lt}/U_{ct}}{F_{lt}} &= \tau_{lt} \end{aligned}$$

Similarly, let the tax rate on capital income for each time $t > 0$ be defined via:

$$
\begin{aligned}
U_{ct} &= \beta U_{ct+1} \left\{ 1 + (F_{kt+1} - \delta)(1 - \tau_{kt+1}) \right\} \\
\tau_{kt+1} &= 1 - \frac{U_{ct}/\beta U_{ct+1} - 1}{(F_{kt+1} - \delta)}
\end{aligned}
$$

As per the consumer's and final goods producer's FOCs , these tax rates will clearly be consistent with utility and profit maximization.

Let the tax on emissions be given as:

$$
\tau_{Et} = p_{Et} - \frac{F_{1lt}}{F_{2lt}}
$$

Again, this tax is clearly consistent with profit maximization in the energy and final goods production sectors as per FOCs (A.6) and (A.5).

To construct bond holdings in period t, first multiply the consumer budget constraint (3) by its Lagrange multiplier γ_t and sum over all periods *from period t onwards*:

$$
\sum_{s=t}^{\infty} \gamma_s \left[C_s + \rho_s B_{s+1} + K_{t+1} - w_s(1 - \tau_{ls})L_s - \left\{ 1 + (r_s - \delta)(1 - \tau_{ks}) \right\} - B_s - \Pi_s - \Omega_t \right] = 0
$$

In a competitive equilibrium, the consumer's FOCs and transversality conditions (A.9) must hold, implying that all future terms relating to capital and bond holdings cancel out. We are thus left with:

$$
\sum_{s=t}^{\infty} \gamma_s \left[C_s - w_s(1 - \tau_{ls})L_s - \Pi_s - \Omega_t \right] + \gamma_t \left\{ 1 + (r_t - \delta)(1 - \tau_{kt}) \right\} K_t = \gamma_t B_t \tag{A.15}
$$

Once again, we can use the agent's and the firms' FOCs to substitute out prices in equation (A.15) and obtain:[52]

$$
\sum_{s=t}^{\infty} \frac{\beta^{s-t} U_{cs}}{U_{ct}} \left[C_s + \frac{U_{ls}}{U_{cs}} L_s - \Omega_s \right] + \frac{U_{ct-1}}{\beta U_{ct}} K_t = B_t
$$

This equation defines the unique bond holdings that are consistent with a competitive equilibrium, given allocations.

Being based on agents' and firms' first order conditions and constraints, the prices and policies

---

[52] For the capital return in period t, note that the substitution derives from:

$$
\begin{aligned}
& \gamma_t \left[r_{kt}(1 - \tau_{kt}) + (1 - \delta) \right] K_t \\
={} & \beta^t U_{ct} \left[\frac{U_{ct-1}}{\beta U_{ct}} \right] K_t \\
={} & \beta^{t-1} U_{ct-1} K_t
\end{aligned}
$$

defined above are clearly consistent with utility and profit maximization. It remains to be shown that all the necessary constraints for competitive equilibrium are satisfied.

The final goods resource constraint, the carbon cycle constraint, the energy production resource constraints, and the factor market clearing conditions all hold by assumption. By Walras' law, demonstrating that the consumer budget constraint is satisfied, is sufficient to imply that the government budget constraint must be satisfied also.

Note that only the consumer's competitive equilibrium-budget constraint is relevant to our proof, as we seek to demonstrate that our constructed prices, bond holdings, and policies are constitute a competitive equilibrium.

In a competitive equilibrium, the household's intertemporal budget constraint must hold, along with the consumer's FOCs, implying (A.7) and (A.8), and the consumer's transversality conditions. The key point, then, is that, at the prices selected above, the consumer's competitive equilibrium-budget constraint then becomes identical to the implementability constraint, which holds by assumption. Thus, the competitive equilibrium budget constraint at the chosen prices is satisfied □.

A.3   Proof of Proposition 4

Proof   Proof in four steps, closely following Atkeson, Chari, and Kehoe (1999).

Step 1: Prove that the upper bound on capital taxes (48) cannot be slack in period t, bind in some period after t, and then become slack again in some period $t + n$ ("Claim 1").

The proof of *Claim 1* proceeds by contradiction. First, note that, with utility of the assumed forms, for $t > 0$,

$$\frac{W_{ct}}{W_{ct+1}} = \frac{U_{ct}}{U_{ct+1}} \tag{A.16}$$

If the constraint (48) is binding for periods $t + 1$ through $t + n$, then for $j \in \{0, ..., n - 1\}$,

$$\frac{U_{ct+j}}{U_{ct+j+1}} = \beta(1 - \delta) \tag{A.17}$$

Combining equations (A.16) and (A.17), and iterating forward yields:

$$\frac{W_{ct+1}}{W_{ct+n}} = \beta^{n-1}(1 - \delta)^{n-1} \tag{A.18}$$

Let Ψ_t denote the Lagrange multiplier on constraint (48) in period t. The planner's FOC with respect to consumption for $t > 0$ in the constrained problem is given by:

$$W_{ct} - \lambda_{1t} + \Psi_t U_{cct} - \Psi_{t-1} U_{cct}(1 - \delta) = 0 \tag{A.19}$$

If $[\Psi_t = \Psi_{t+n} = 0]$ and $[\Psi_{t+1}, \Psi_{t+2}, ..., \Psi_{t+n-1} > 0]$, then based on (A.19),

$$\lambda_{1t} = W_{ct} - (1 - \delta)\Psi_{t-1} U_{cct} \tag{A.20}$$

$$\lambda_{1t+1} = W_{ct+1} + \Psi_{t+1} U_{cct+1} \tag{A.21}$$

and

$$\lambda_{1t+n} = W_{ct+n} - (1 - \delta)\Psi_{t+n-1}U_{cct+n} \tag{A.22}$$

The planner's FOC for K_{t+1} is unchanged and implies that:

$$\lambda_{1t} = \beta\lambda_{1t+1}\left[F_{kt+1} + (1 - \delta)\right] \tag{A.23}$$

From the planner's FOC for capital (A.23), we further can infer that:

$$\lambda_{1t} = \beta\lambda_{1t+1}[(1 - \delta) + F_{kt+1}] \geq \beta\lambda_{1t+1}(1 - \delta)$$

Plugging in from equations (A.20)-(A.22), and iterating forward yields:

$$W_{ct+1} + \Psi_{t+1}U_{cct+1} \geq \beta^{n-1}(1 - \delta)^{n-1}[W_{ct+n} - (1 - \delta)\Psi_{t+n-1}U_{cct+n}] \tag{A.24}$$

Finally, plugging in from equation (A.18) into condition (A.24) results in the contradiction required to prove *Claim 1*:

$$W_{ct+n}\beta^{n-1}(1 - \delta)^{n-1} + \Psi_{t+1}U_{cct+1} \geq \beta^{n-1}(1 - \delta)^{n-1}[W_{ct+n} - (1 - \delta)\Psi_{t+n-1}U_{cct+n}] \tag{A.25}$$
$$\Psi_{t+1}U_{cct+1} \geq \beta^{n-1}(1 - \delta)^{n-1}[-(1 - \delta)\Psi_{t+n-1}U_{cct+n}]$$

Since we have assumed that $\Psi_{t+1} > 0$ and $\Psi_{t+n-1} > 0$, condition (A.25) implies a contradiction since $U_{cc_t} < 0$.

Step 2: Show that Ψ_t cannot be positive in every period.

Proof by contradiction. As argued by Atkeson, Chari, and Kehoe (1999), suppose that Ψ_t was binding in every period, implying that the household would always be indifferent to just holding his capital stock and letting it depreciate (rather than investing it). In that case, the capital stock would go zero at rate $K_{t+1} = (1 - \delta)K_t$. However, given the assumption that $F(0, L_t, E_t) = 0$, this would violate the resource constraint. Hence, the constraint cannot bind in every period.

Step 3: Show that, if t is the last period in which (48) binds, the optimal capital tax may be at an intermediate value in period $t + 1$, but is zero in all periods on or after $t + 2$.

Consider the last period t in which the upper bound binds. We then know that $\Psi_{t+1} = 0$, and hence, given the planner's FOC for consumption (A.19), we know that, for $t + 1$,

$$\lambda_{1t+1} = W_{ct+1} - (1 - \delta)\Psi_t U_{cct+1} \tag{A.26}$$

and for $s \geq t + 2$,

$$\lambda_{1s} = W_{cs} \tag{A.27}$$

Combining (A.27) with the optimality condition for capital (A.23), implies that:

$$W_{cs} = \beta W_{cs+1}[(1 - \delta) + F_{ks+1}]$$

and since

$$W_{ct}/W_{ct+1} = U_{ct}/U_{ct+1}$$

this implies an optimal capital income tax of zero for $s \geq t + 2$.

Step 4:

I now consider the implications of *Claims 1-3* for optimal carbon tax component to internalize output damages. Whether this tax is greater or less than Pigouvian depends on whether:

$$\sum_{j=0}^{\infty} \beta^j \frac{\lambda_{1t+j}}{\lambda_{1t}} \left[\frac{\partial Y_{t+j}}{\partial T_{t+j}} \frac{\partial T_{t+j}}{\partial E_t} \right] \lessgtr \sum_{j=0}^{\infty} \beta^j \frac{U_{ct+j}}{U_{ct}} \left[\frac{\partial Y_{t+j}}{\partial T_{t+j}} \frac{\partial T_{t+j}}{\partial E_t} \right]$$

Whenever Ψ_t is binding, we know that:

$$\frac{\beta U_{ct+1}}{U_{ct}} = (1 - \delta)^{-1}$$

So the question is whether, at those times,

$$\frac{\beta \lambda_{1t+1}}{\lambda_{1t}} \lessgtr (1 - \delta)^{-1} = \frac{\beta U_{ct+1}}{U_{ct}}$$

From the planner's FOC for capital (A.23), we know that:

$$\frac{\beta \lambda_{1t+1}}{\lambda_{1t}} = [(1 - \delta) + F_{kt+1}]^{-1} < (1 - \delta)^{-1}$$

where the inequality follows from the assumption that capital is an essential input to production. Hence:

$$\frac{\beta \lambda_{1t+1}}{\lambda_{1t}} < \frac{\beta U_{ct+1}}{U_{ct}}$$

We thus see that, if Ψ_t is binding for at least period t, the optimal carbon tax is less than Pigouvian. The intuition is that a less-than-Pigouvian tax is equivalent to a positive capital income tax for the climate-damage based intertemporal margin.

The issue left to be determined is what happens if period t is the intermediate period, when the upper bound was binding before and the optimal capital tax is zero from period $t+1$ onwards.

Combining the planner's FOCs for consumption (A.26) and (A.27) for period $s = 1 + t$ yields:

$$\frac{\lambda_{1t+1}}{\lambda_{1t}} = \frac{W_{ct+1}}{W_{ct} - \beta^{-1} \Psi_{t-1} U_{cct}(1 - \delta)} < \frac{W_{ct+1}}{W_{ct}} = \frac{U_{ct+1}}{U_{ct}}$$

where the inequality follows from the fact that $[U_{cct} < 0]$, and the second equality follows from the assumption on the structure of preferences (49)-(50). Hence, we also find a less-than-Pigouvian tax on output damages in period t when the last period in which the constraint was binding was $t - 1$.

Overall, we thus shown that the optimal carbon tax to internalize output damages is less-than-Pigouvian for a finite number of periods, and jumps to the Pigouvian level as soon as capital income taxes are optimally set to zero.

A.4 Exogenously Fixed Capital Tax Rates

The planner's problem is now given by (16) with the addition of the capital tax constraint (51):

$$\max_k \sum_{t=0}^{\infty} \beta^t [U(C_t, L_t, T_t) + \underbrace{\phi[U_{ct}C_t + U_{lt}L_t]]}_{\equiv W_t}$$

$$-\phi\{U_{c0}[K_0\{1 + (F_{k0} - \delta)(1 - \tau_{k_0})\}]\}$$

$$+\sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[\left\{ A_t(T_t)\widetilde{F_{1t}}(L_{1t}, E_t, K_{1t}) \right\} + (1 - \delta)K_t - C_t - G_t - K_{t+1} \right]$$

$$+\sum_{t=0}^{\infty} \beta^t \xi_t [T_t - \mathcal{F}(S_0, E_0, E_1, ...E_t)]$$

$$+\sum_{t=0}^{\infty} \beta^t \lambda_{lt} [L_t - L_{1t} - L_{2t}]$$

$$+\sum_{t=0}^{\infty} \beta^t \lambda_{kt} [K_t - K_{1t} - K_{2t}]$$

$$+\sum_{t=0}^{\infty} \beta^t \omega_t [F_{2t}(A_{Et}, K_{2t}, L_{2t}) - E_t]$$

$$-\sum_{t=0}^{\infty} \beta^t \Psi_t \left[\underbrace{\frac{U_{ct}}{\beta U_{ct+1}} - [1 + (1 - \overline{\tau_k})(F_{kt+1} - \delta)]}_{\equiv \chi_t} \right]$$

The planner's first order condition with respect to temperature change T_t after $t > 0$ implies the following marginal welfare cost of temperature change in period t, ξ_t :

$$-U_{Tt} - \lambda_{1t}F_{Tt} + \frac{1}{\beta}\Psi_t\chi_{Tt-1} = \xi_t$$

Here, Ψ_t denotes the Lagrange multiplier on the capital income tax constraint, and χ_{Tt-1} reflects the derivative of the capital tax constraint with respect to temperature change at time. The marginal welfare cost of temperature change thus consists of utility damages U_{Tt}, production damages F_{Tt} (valued at the *public* marginal utility of income λ_{1t}), plus an additional term reflecting the degree to which temperature change relaxes or tightens the capital tax constraint.

Note that:

$$\chi_{Tt} = (-1)\frac{\partial^2 F_{1t}}{\partial K_t \partial T_t}$$

If the government would ideally set capital taxes below $\overline{\tau_k}$, $\Psi_t > 0$, and since we are assuming that T_t negatively affects all marginal products, $\chi_{Tt} > 0$, and hence marginal welfare costs of temperature change are *higher* than without the capital tax constraint. Intuitively, this is because temperature change decreases the marginal product of capital, and thus exacerbates the capital income tax constraint.

Next, the FOC for energy use E_t for $t > 0$:

$$\lambda_{1t} F_{Et} - \sum_{t=0}^{\infty} \xi_{t+j} \frac{\partial T_{t+j}}{\partial E_t} - \frac{1}{\beta} \Psi_t \chi_{Et-1} = \omega_t \tag{A.28}$$

The planner thus seeks to equate the marginal cost of energy production ω_t with its marginal benefit in final goods production F_{Et}, adjusted for the present value of marginal costs with the associated temperature change $\sum_{t=0}^{\infty} \xi_{t+j} \frac{\partial T_{t+j}}{\partial E_t}$, and, in this setting, for the way in which energy use affects the capital income tax constraint, $\Psi_t \chi_{Et-1}$. Note that:

$$\chi_{Et} = (-1)(1 - \overline{\tau_k}) \frac{\partial^2 F_{1t}}{\partial K_t \partial E_t}$$

If capital and energy are complements in final goods production, $\chi_{Et} < 0$. If the planner would ideally want to set capital taxes below $\overline{\tau_k}$, $\Psi_t > 0$, and the marginal benefit of energy production is adjusted *upwards* due to its impacts on the capital tax constraint in (A.28). Intuitively, since higher energy uses increases the marginal product of capital, it helps counteract the exogenously given capital income tax. As a result, energy use is more valuable to the planner, ceteris paribus.

Finally, the marginal cost of energy production ω_t is now also adjusted to reflect the impact of changes in labor supply L_t and its allocation between the two production sectors (L_{1t}, L_{2t}) on the capital tax constraint. Combining the corresponding first order conditions yields:

$$\omega_t = \lambda_{1t} \underbrace{\frac{F_{l1t}}{F_{2lt}}}_{\substack{\text{Private MC of} \\ \text{energy production}}} - \underbrace{\frac{\Psi_{t-1} \chi_{l1t-1}}{\beta F_{2lt}}}_{\substack{\text{Capital tax} \\ \text{constraint inter-} \\ \text{action adjustment}}}$$

where:

$$\chi_{l1t} = (-1)(1 - \overline{\tau_k}) \frac{\partial^2 F_{1t}}{\partial K_{1t} \partial L_{1t}}$$

If capital and labor are complements in final goods production then $\chi_{l1t} < 0$. If the planner would want to set capital taxes below $\overline{\tau_k}$, $\Psi_t > 0$, and the marginal cost of energy production is thus adjusted *upwards* to reflect the decrease in the marginal product of capital (and thus the tightening of the capital income tax constraint) associated with allocating labor away from final goods production and towards energy production.

Combining the planner's first order conditions leads to the following implicit expression for optimal carbon taxes in this setting, conditional on all other taxes being set optimally (given the constraints):

$$
\begin{aligned}
\tau_{Et} &= \sum_{t=0}^{\infty} \left[\underbrace{\frac{U_{Tt+j}}{\lambda_{1t}}}_{\text{Utility damages}} + \underbrace{\frac{\lambda_{1t+j}}{\lambda_{1t}} F_{Tt+j}}_{\text{Output damages}} - \underbrace{\frac{1}{\beta} \frac{\Psi_{t+j}}{\lambda_{1t}} \chi_{Tt-1+j}}_{\substack{\text{Temperature change} \\ \text{impact on } \overline{\tau_k} \text{ constraint}}} \right] \frac{\partial T_{t+j}}{\partial E_t} \\
&\quad - \underbrace{\frac{1}{\beta} \frac{\Psi_{t-1}}{\lambda_{1t}} \chi_{Et-1}}_{\substack{\text{Energy use} \\ \text{impact on } \overline{\tau_k} \text{ constraint}}} + \underbrace{\frac{1}{F_{2lt}} \frac{1}{\beta} \frac{\Psi_{t-1}}{\lambda_{1t}} \chi_{l1t-1}}_{\substack{\text{Energy production cost} \\ \text{impact on } \overline{\tau_k} \text{ constraint}}}
\end{aligned}
$$

There are thus several countervailing forces affecting the optimal carbon tax formulation in a setting with exogenously given, suboptimal capital income tax rates. Which effect dominates is ex ante ambiguous. Contrary to the endogenously arising capital income tax in Proposition 4, when the capital tax is imposed exogenously, the planner may thus adjust optimal carbon taxes upwards or downwards.

A.5 Nonrenewable Resource Setting

The first goal is to derive the alternative implementability constraint (54). First, based on the fossil fuel producer's FOC (52), one can write the private shadow price of the resource at time zero $\widetilde{\mu}_0$ as:

$$
\widetilde{\mu}_0 = p_{E0} - \tau_{E0} \tag{A.29}
$$

Substituting in for $p_{E0} = F_{E0}$ from the final goods producer's FOCs (8), and remembering that $\overline{\tau_{E0}}$ is exogenously given by assumption transforms expression (A.29) into an object that depends only on the allocations and parameters:

$$
\widetilde{\mu}_0 = p_{E0} - \overline{\tau_{E0}}
$$

Next, iterating this expression forward based the firm's Hotelling condition (53), yields:

$$
\widetilde{\mu}_t = \frac{\widetilde{\mu}_0}{q_t} \tag{A.30}
$$

Note that the evolution of prices of the consumption good over time in a competitive equilibrium necessarily satisfies:[53]

$$q_t = \frac{\beta^t U_{ct}}{U_{c0}} \tag{A.31}$$

Combining equations (A.29)-(A.31) leads to an expression for Hotelling rents in period t as a function of only the allocations:

$$\widetilde{\mu}_t = \frac{(U_{c0})(F_{E0} - \overline{\tau_{E0}})}{\beta^t U_{ct}} \tag{A.32}$$

Finally, based on the fossil fuel producer's optimality condition (52), we can thus express profits as a function of only the allocations:

$$\begin{aligned} \Pi_t &= [p_{Et} - \tau_{Et}] E_t \\ &= [\widetilde{\mu}_t] E_t \\ &= \left[\frac{U_{c0}}{\beta^t U_{ct}} (F_{E0} - \overline{\tau_{E0}})\right] E_t \end{aligned} \tag{A.33}$$

As formally demonstrated in the proof of proposition 1, the generic implementability constraint for this economy is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}C_t + U_{lt}L_t - U_{ct}\Pi_t\right] \leq U_{c0}\left[K_0\left\{1 + (F_{k0} - \delta)(1 - \tau_{k_0})\right\} + B_0\right] \tag{A.34}$$

In the core version of the model, $\Pi_t = 0$. However, in the nonrenewable resource setting, one can substitute in for Π_t from (A.33) to obtain the desired implementability constraint (54):

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}C_t + U_{lt}L_t\right] = U_{c0}\left[K_0\left\{1 + (F_{k0} - \delta)(1 - \tau_{k_0})\right\} + B_0 + \sum_{t=0}^{\infty} E_t\left(F_{E0} - \overline{\tau_{E0}}\right)\right]$$

---

[53]

Consider setting up the consumer's problem as subject to a single present value budget constraint:

$$\sum_{t=0}^{\infty} q_t \left(c_t + K_{t+1} + \rho B_{t+1} - w_t(1 - \tau_{lt})L_t - r_t(1 - \tau_{kt})K_t - (1 - \delta)K_t + \Pi_t\right) = 0$$

Then, for γ as Lagrange multiplier on this constraint, the FOC for consumption yields:

$$\beta^t U_{ct} = \gamma q_t$$

Combining the FOCs for C_t and C_{t+1} leads to:

$$\frac{q_t}{q_{t+1}} = \frac{U_{ct}}{\beta U_{ct+1}}$$

Setting $q_0 = 1$ as the numeraire and iterating to period 0 thus leaves us with the desired result.

A.5.1 Proposition 6

The planner's problem in this economy is given by:

$$\max_k \underbrace{\sum_{t=0}^{\infty} \beta^t U(C_t, L_t, T_t) + \phi \sum_{t=0}^{\infty} \beta^t \left[U_{ct} C_t + U_{lt} L_t \right]}_{\equiv W_t} \tag{A.35}$$

$$+ \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[\left\{ (1 - D(T_t))) \cdot \widetilde{F}_t(L_t, E_t, K_t) \right\} + (1 - \delta) K_t - C_t - G_t - K_{t+1} \right]$$

$$+ \sum_{t=0}^{\infty} \beta^t \xi_t [T_t - \mathcal{F}(S_0, E_0, E_1, ... E_t)]$$

$$+ \sum_{t=0}^{\infty} \beta^t \mu_t \left[R_t - E_t - R_{t+1} \right]$$

$$- \phi U_{c0} \left[K_0 \left\{ 1 + (F_{k0} - \delta)(1 - \tau_{k_0}) \right\} + B_0 + \sum_{t=0}^{\infty} E_t \left(F_{E0} - \overline{\tau_{E0}} \right) \right]$$

Combining the planner's first order conditions for energy use E_t, the fossil resource stock R_{t+1}, and for temperature change T_t for $t > 0$ yields the following law of motion:

$$F_{Et} - \sum_{j=0}^{\infty} \beta^j \left[\frac{U_{Tt+j}}{\lambda_{1t}} + \frac{\lambda_{1t+j}}{\lambda_{1t}} \frac{\partial F_{1t+j}}{\partial T_{t+j}} \right] \frac{\partial T_{t+j}}{\partial E_t} - \frac{\phi U_{c0}}{\beta^t \lambda_{1t}} \left(F_{E0} - \overline{\tau_{E0}} \right) \tag{A.36}$$

$$= \beta \frac{\lambda_{1t+1}}{\lambda_{1t}} \left[F_{Et+1} - \sum_{j=0}^{\infty} \beta^j \left[\frac{U_{Tt+1+j}}{\lambda_{1t+1}} + \frac{\lambda_{1t+1+j}}{\lambda_{1t+1}} \frac{\partial F_{1t+1+j}}{\partial T_{t+1+j}} \right] \frac{\partial T_{t+1+j}}{\partial E_{t+1}} - \frac{\phi U_{c0}}{\beta^{t+1} \lambda_{1t+1}} \left(F_{E0} - \overline{\tau_{E0}} \right) \right]$$

When $\frac{\lambda_{1t+1}}{\lambda_{1t}} \neq \frac{U_{ct+1}}{U_{ct}}$, it is difficult to infer analytically what carbon tax could decentralize (A.36) along the optimal policy path. However, in the benchmark case where preferences satisfy (49) or (50), it is easy to show that:

$$\frac{\lambda_{1t+1}}{\lambda_{1t}} = \frac{W_{ct+1}}{W_{ct}} = \frac{U_{ct+1}}{U_{ct}} \tag{A.37}$$

where the first equality follows from the planner's first order conditions for consumption for $t > 0$. In this case, (A.36) becomes:

$$F_{Et} - \sum_{j=0}^{\infty} \beta^j \left[\left(\frac{1}{MCF_t} \right) \frac{U_{Tt+j}}{U_{ct}} + \frac{U_{ct+j}}{U_{ct}} \frac{\partial F_{1t+j}}{\partial T_{t+j}} \right] \frac{\partial T_{t+j}}{\partial E_t} - \left[1 - \frac{1}{MCF_t} \right] \frac{\widetilde{\mu}_t}{\kappa} \tag{A.38}$$

$$= \beta \frac{U_{ct+1}}{U_{ct}} \left[F_{Et+1} - \sum_{j=0}^{\infty} \beta^j \left[\left(\frac{1}{MCF_{t+1}} \right) \frac{U_{Tt+1+j}}{U_{ct+1}} + \frac{U_{ct+1+j}}{U_{ct+1}} \frac{\partial F_{1t+1+j}}{\partial T_{t+1+j}} \right] \frac{\partial T_{t+1+j}}{\partial E_{t+1}} - \left[1 - \frac{1}{MCF_{t+1}} \right] \frac{\widetilde{\mu_{t+1}}}{\kappa} \right]$$

To see why this is the case, first apply (A.37) to (A.36) for all j. Next, multiply utility damages on each side by U_{ct}/U_{ct} and U_{ct+1}/U_{ct+1}, respectively, and invoked the definition of the marginal cost of public funds (17). Finally, note that the assumption that preferences satisfy (49) or (50) implies that the Lagrange multiplier on the implementability constraint, ϕ, can be re-written as:

$$\phi = \frac{[MCF_t - 1]}{\kappa}$$

where the constant $\kappa = (1 - \sigma)$ for (49) and $\kappa = (1 - \sigma)(1 - \gamma)$ for (50).[54] Substituting into (A.36), multiplying each side by U_{ct}/U_{ct} and U_{ct+1}/U_{ct+1}, respectively, and invoking the definition of the Hotelling rent $\widetilde{\mu}_t$ at time t (A.32) leads to expression (A.38).

Finally, consider the fossil fuel producer's optimality condition (53),

$$(p_{Et} - \tau_{Et}) = \frac{q_{t+1}}{q_t} (p_{Et+1} - \tau_{Et+1})$$

and substitute in for prices from the final goods producer and consumer first order conditions, yielding:

$$F_{Et} - \tau_{Et} = \left(\beta \frac{U_{ct+1}}{U_{ct}} \right) (F_{Et+1} - \tau_{Et+1}) \tag{A.39}$$

Proposition 6 follows immediately from a comparison of (A.39) and (A.38), and invoking the definitions of Pigouvian taxes in (19) and (20).

If preferences are of the form (49) or (50), and if profit taxes are not available, the optimal carbon tax at time $t > 0$ is implicitly defined by:

$$\tau_{Et}^* = \left(\frac{\tau_{Et}^{Pigou,U}}{MCF_t} \right) + \tau_{Et}^{Pigou,Y} + \left[1 - \frac{1}{MCF_t} \right] \frac{\widetilde{\mu}_t}{\kappa}$$

B   Appendix B

B.1   Marginal Cost of Public Funds Survey

Table 6 below summarize estimates from the literature on the marginal cost of public funds and the closely related concepts of the marginal excess burden (MEB), and the marginal welfare cost (MWC) of taxes. In order to infer the plausible magnitude of a global MCF estimate, I proceed as follows.

---

[54]   This follows from the planner's first order condition for consumption for $t > 0$,

$$U_{ct} + \phi (U_{cct} C_t + U_{ct} + U_{lct} L_t) = \lambda_{1t}$$

where one can substitute in for the partial derivatives of the utility function and rearrange.

First, for studies reporting a range of estimates, I use the authors' preferred or central estimate when available. Otherwise, the mean estimate is used (See table for details).

Second, for studies reporting MCF estimates for multiple tax instruments and for all taxes, I use the "all taxes" value. For studies reporting several estimates across different tax instruments but no overall estimate, each estimate is used as separate observation and averaged along with the other estimates.

Third, for studies that estimate the marginal excess burden (MEB) or the marginal deadweight loss ($MDWL$) from taxes, I use $1 + MEB$ and $1 + MDWL$ as measures of the MCF. Please note that the results in Table 6 are reported with (+1) added for those studies. As discussed in Section 2, this is not technically accurate due to differences in the precise definitions. However, in reality, there are considerable differences in definitions and calculation methods across studies even within each measurement concept (see, e.g., Jorgenson and Yun (1991) on discussion of MEB measurement differences), implying that estimates across studies may in any case not be perfectly comparable.

Fourth, I compute unweighted averages across estimates within each country. The base year (2005) PPP GDP-weighted average of the central estimates across countries is 1.486.[55]

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[55] Year 2005 GDP data stem from the World Bank Development Indicators data base.

Authors	Data	Results	Notes
Saez, Slemrod, and Giertz (2012)	U.S.	1.195	MEB (across-the-board proportional tax increase)
Parry (2002)	U.S.	1.27; 1.39	MWC (labor income), "middle" parameters, spending on public goods/transfers
Jorgenson and Yun (2001)	U.S.	1.266	MEB (all taxes)
Feldstein (1999)	U.S.	2.06	MCF (personal income)
Ahmed and Croushore (1994)	U.S.	1.121 to 1.167	MCF (labor income)
Jorgenson and Yun (1991)	U.S.	1.391	MEB (all taxes)
Fullerton and Henderson (1989)	U.S.	1.247	MEB (personal income), "standard elasticities"
Fullerton and Henderson (1989)	U.S.	1.310	MEB (corporate income), "standard elasticities"
Browning (1987)	U.S.	1.318; 1.469	MWC (labor income), author's preferred estimates
Ballard, Shoven, and Whalley (1985)	U.S.	1.332	MEB (all taxes), value for parameters in which authors have "most confidence"
Stuart (1984)	U.S.	1.41	MEB (labor income), average across all estimates reported in Table 2
Kleven and Kreiner (2006)	UK	1.26	MCF (labor income), proportional tax increase, "natural baseline scenario" S6
Kleven and Kreiner (2006)	Italy	1.52	MCF (labor income), proportional tax increase, "natural baseline scenario" S6
Kleven and Kreiner (2006)	France	1.72	MCF (labor income), proportional tax increase, "natural baseline scenario" S6
Kleven and Kreiner (2006)	Germany	1.85	MCF (labor income), proportional tax increase, "natural baseline scenario" S6
Kleven and Kreiner (2006)	Denmark	2.20	MCF (labor income), proportional tax increase, "natural baseline scenario" S6
Hansson and Stuart (1985)	Sweden	1.55	MCF (labor income), average of "best guess" parameter estimates for spending on public goods/ transfers, across historical and const.progr. tax increase
Baylor and Beausejour (2004)	Canada	1.32	MWC (personal income), "central" parameter values
Baylor and Beausejour (2004)	Canada	1.37	MWC (corporate income), "central" parameter values
Ruggeri (1999)	Canada	1.13	MCF (personal income), proportional increase
Thirsk and Moore (1991)	Canada	1.30 to 1.43	MWC (labor income), range for "intermediate parameters"
Ahmad and Stern (1987)	India	1.66-2.15	MCF (excise taxes) (as reported by Auriol and Warlters, 2012)
Ahmad and Stern (1987)	India	1.59-2.12	MCF (sales taxes) (as reported by Auriol and Warlters, 2012)
Ahmad and Stern (1987)	India	1.54-2.17	MCF (import taxes) (as reported by Auriol and Warlters, 2012)
Auriol and Warlters (2012)	S.S Africa	1.21	MCF (all taxes), "base case estimate" of average across 38 countries
Diewert and Lawrence (2002)	Australia	1.48	MEB (capital income), final model year (1994) value
Campbell and Bond (1997)	Australia	1.194;1.239	MCF (labor income)

Table 6: MCF Literature Review

B.2 Energy Production Function Labor Share Estimation

This section describes the estimation of the labor share in carbon-based energy production. Industry data from the U.S. Bureau of Economic Analysis ("BEA") on *components of value added by industry* were used for this calculation. Two technical points deserve special attention. First, well-known problems arise with regards to the treatment of mineral resources in industry and national accounts (BEA, 1994). Resource rents are not accounted for explicitly and are thus included as capital returns. Given this concern, and given that the baseline model and calibration focus on carbon energy in sufficiently large supply so as to not earn Hotelling rents (e.g., coal), I thus focus on data from the non-oil and gas energy industries as listed below. Second, in using the BEA data, it is necessary to distribute proprietors' income between capital or labor. In each of the industries considered, base year proprietors' income shares of value added are small, between 4.2% and 5.4%. I follow Valentyni and Herrendorf (2007) in calculating capital and labor shares without proprietors' income. This approach assumes that proprietor's income is split between capital and labor in the same way as other income.[56]

Table 7 summarizes the results from these factor elasticity calculations.

Industry Title	2002 NAICS	2000-2010 Average: Labor Share	2000-2010 Average: GDP Share
Mining, except oil and gas	212	0.606	0.0029
Support activities for mining	213	0.641	0.0024
Utilities	22	0.382	0.0175
Manufacturing of petroleum and coal products	324	0.181	0.0084
All Private Industries		0.719	
Weighted Average Share:		0.368	
Weighted Average Share w/o petroleum/coal manufacturing:		0.438	

Table 7: Labor Share of Value Added in Energy Production

A labor share value of $\alpha_E = 0.403$ is used a compromise between the estimates with and without petroleum and coal products manufacturing, respectively.

---

[56] Very specifically, labor shares are calculated via:

$$\widehat{\alpha_E} = \frac{COM}{COM + \{GOS - BTP - PROP\}}$$

where COM is compensation of employees (including employer contributions to pensions, etc.), GOS is gross operating surplus, BTP is net business current transfer payments, and $PROP$ is proprietors' income, measured in the data as "Other gross operating surplus, noncorporate."

B.3 Appendix: Preferences, Balanced Growth, and the Elasticity of Labor Supply

B.3.1 Preferences Calibration

The Frisch elasticity of labor supply $\left(\eta^F = \frac{\partial l_t}{\partial w_t} \frac{w_t}{l_t} |_{\lambda_t} \right)$ in the current setting is easily derived to equal:

$$\eta^F = \frac{U_{lt}}{l_t \left[U_{llt} - \frac{U_{clt}^2}{U_{cct}} \right]} \tag{B.1}$$

$$= \frac{(1 - \phi l_t)}{\phi l_t} \frac{-1}{\left[[\gamma(1-\sigma) - 1] - \frac{\gamma(1-\sigma)^2}{(-\sigma)} \right]} \tag{B.2}$$

Similarly, the representative household's first oder condition for labor supply is given by:

$$\frac{-U_{lt}}{U_{ct}} = w_t(1 - \tau_{lt})$$

$$\frac{c_t \gamma \phi}{(1 - \phi n_t)} = w_t(1 - \tau_{lt}) \tag{B.3}$$

I use the two equations (B.2) and (B.3) to solve for the two unknowns γ and ϕ as a function of η^F, l_t, c_t, $(1 - \tau_{lt})w_t$, and σ. I calibrate to $t = 2005$ values from the data. The choice to calibrate to the base year is made to increase consistency across model runs with different fiscal scenarios. That is, steady-state labor supply depends on the steady-state labor income tax rate, which is an endogenous outcome of the model and can differ across the fiscal scenarios considered. Differences in steady-state labor supply would then require differences in preference parameters across model runs. These changes would obfuscate the interpretation of the results as being due to changes in tax policy and constraints across model scenarios. Observed base-year values for consumption and labor supply are given from the data and have the attractive trait of being constant across fiscal scenarios. An important exception is the calibration to the first-best (lump sum taxation) setting, which sets $(\tau_{l2005} = 0)$.

Baseline labor supply l_{2005} is estimated using OECD data on "Average annual hours actually worked per worker" and on employment rates across all available countries in the model base year 2005. Given Jones, Manuelli, and Rossi's (1993) assumption that adults have 14.5 hours per day available for work, the GDP-weighted average time endowment share spent on labor is $l_{2005} = 0.2272$. Base year consumption per capita c_{2005} is calculated using World Bank data[57] on household final consumption expenditure as share of GDP across all available countries, which is 61% for 2005.[58] The gross wage w_{2005} is calculated as the marginal product of labor in the base year $\left(w_t = \frac{(1-\alpha-v)Y_{2005}}{l_{2005}N_{2005}} \right)$. The base year average marginal labor tax rate τ_{l2005} is the GDP-

---

[57] *World Development Indicators* data base, World Bank.

[58] An alternative calculation based on the assumed government consumption expenditure share (17%), discussed in Section 4.7, and the optimal initial year savings rate implied by most R-CEM model runs (24%) yield a very similar figure of 59%.

weighted average labor-consumption effective tax based on estimates from Carey and Rabesona (2002) as discussed in section 4.7. Finally, the value $\sigma = 1.5$ is chosen to match the DICE model (Nordhaus, 2010). The resulting estimates in all distortionary tax model runs are $\gamma = 0.679$ and $\phi = 2.25$ (such that "utility-effective" labor $\phi l_{2005} = 0.511$). In the first-best model run, $\tau_{l2005} = 0$ but all other parameters remain the same, yielding $\gamma = 1.195$ and $\phi = 2.105$ (with $\phi l_{2005} = 0.478$).

B.3.2 Demonstrate Compatibility with Balanced Growth

Let $\mathcal{L}_t = 1 - l_t$ denote *leisure*. To demonstrate that utility specification

$$
\begin{aligned}
U(c_t, l_t, T_t) &= \left\{ \frac{[c_t \cdot (1 - \phi l_t)^\gamma]^{1-\sigma}}{1 - \sigma} \right\} + \alpha_0 (T_t)^{a_1} \\
&= \left\{ \frac{[c_t \cdot v(\mathcal{L})]^{1-\sigma}}{1 - \sigma} \right\} + \alpha_0 (T_t)^{a_1}
\end{aligned}
$$

is compatible with a balanced growth path for $[\sigma = 1.5 > 1]$, one has to show that (King et al., 2001):

1. $\{v(\mathcal{L})\}^{1-\sigma}$ is decreasing

2. $\{v(\mathcal{L})\}^{1-\sigma}$ is convex

3. $-\sigma v(\mathcal{L}) v''(\mathcal{L}) > (1 - 2\sigma)[v'(\mathcal{L})]^2$

The first step is thus to express $(1 - \phi l_t)$ as a function of leisure $v(\mathcal{L}_t)$:

$$
\begin{aligned}
(1 - \phi n_t) &= \phi + (1 - \phi) - \phi n_t \\
&= \phi(1 - n_t) + (1 - \phi) \\
&= \phi \mathcal{L}_t + (1 - \phi)
\end{aligned}
$$

The first and second derivatives of $v(\mathcal{L}_t)$ are correspondingly given by:

$$
\begin{aligned}
v(\mathcal{L}) &= [\phi \mathcal{L} + (1 - \phi)]^\gamma & \text{(B.4)} \\
v'(\mathcal{L}) &= \gamma \phi [\phi \mathcal{L} + (1 - \phi)]^{\gamma - 1} \\
v''(\mathcal{L}) &= \gamma \phi^2 [\gamma - 1][\phi \mathcal{L} + (1 - \phi)]^{\gamma - 2}
\end{aligned}
$$

(1) Ensure that $\{v(\mathcal{L})\}^{1-\sigma}$ is decreasing:

$$
\begin{aligned}
v(\mathcal{L})^{1-\sigma} &= [\phi \mathcal{L} + (1 - \phi)]^{\gamma(1-\sigma)} & \text{(B.5)} \\
v'(\mathcal{L})^{1-\sigma} &= \gamma \phi (1 - \sigma)[\phi \mathcal{L} + (1 - \phi)]^{\gamma(1-\sigma)-1}
\end{aligned}
$$

Given that $(1 - \sigma) < 0$, expression (B.5) is negative if $\phi > 0$, $\gamma > 0$, and $\phi \mathcal{L} + (1 - \phi) > 0$. A sufficient condition for the latter to be true is that $\phi \in [0, 1]$. For low values of the Frisch

elasticity of labor supply and for $n^* = 0.2272$, in the baseline calibration, there are cases when $\phi > 1$. In those cases, however, $\phi\mathcal{L} + (1 - \phi) > 0$, implying that the conditions for $\{v(\mathcal{L})\}^{1-\sigma}$ being decreasing are still satisfied.

(2) Ensure that $\{v(\mathcal{L})\}^{1-\sigma}$ is convex:

$$v''(\mathcal{L})^{1-\sigma} = \gamma(1-\sigma)\phi^2 \left[\gamma(1-\sigma) - 1\right] \cdot \left[\phi\mathcal{L} + (1-\phi)\right]^{\gamma(1-\sigma)-2} \tag{B.6}$$

Since $(1 - \sigma) < 0$ and $[\gamma(1-\sigma) - 1] < 0$ (for $\gamma > 0$), expression (B.6) will be positive as long as the conditions derived in (1) are satisfied.

(3) Ensure that $-\sigma v(\mathcal{L})v''(\mathcal{L})) > (1 - 2\sigma)[v'(\mathcal{L})]^2$:
Substituting in from (B.4) and collecting terms yields:

$$(-\sigma)\gamma\phi^2[\gamma - 1][\phi\mathcal{L} + (1-\phi)]^{2\gamma-2} > (1 - 2\sigma)\gamma^2\phi^2[\phi\mathcal{L} + (1-\phi)]^{2\gamma-2}$$

which can be reduced to condition:

$$\frac{\sigma}{(1-\sigma)} < \gamma \tag{B.7}$$

Once again, since $\sigma = 1.5$, condition (B.7) is satisfied as long as $\gamma > 0$.

B.3.3 Long-Run Marginal Cost of Public Funds

This section derives an implicit expression for the MCF along a balanced growth path in an optimal tax (Ramsey) equilibrium for the functional forms of the COMET model. Combining the planner's first order conditions with respect to C_t, L_t, and L_{1t} for $t > 0$ and substituting in from the agent's and firm's optimality conditions yields:

$$MCF_t = 1 + \tau_{lt}^* \left[\frac{U_{cct}C_t + U_{ct} + U_{lct}L_t + U_{cct}(\Omega_t)}{\left\{ [U_{cct}C_t + U_{ct} + U_{lct}L_t - U_{cct}(\Omega_t)] + \frac{1}{F_{lt}} [U_{clt}C_t + U_{lt} + U_{llt}L_t - U_{clt}(\Omega_t)] \right\}} \right] \tag{B.8}$$

Note that (B.8) differs from formulation (18) by accounting for government transfers to households, Ω_t. Taking these derivatives for preferences (41), rearranging, and plugging in based on the agent's and firm's first order conditions allows one to re-express (B.8) as:

$$1 + \tau_{lt} \left[\frac{B}{\left\{ B - (1 - \tau_{lt}) \left[(1-\sigma) + 1 + [\gamma(1-\sigma) - 1] \left[\frac{1-\phi l_t}{\phi l_t} \right]^{-1} + (1-\sigma)\frac{\Omega_t}{C_t} \right] \right\}} \right] \tag{B.9}$$

where:

$$B = (1-\sigma) + \gamma(1-\sigma) \left[\frac{1 - \phi l_t}{\phi l_t} \right]^{-1} - \sigma\frac{\Omega_t}{C_t}$$

Let s_{ss} denote the fraction of output devoted to gross investment on a balanced growth path. In

the long run, when the backstop energy technology is assumed to be cost competitive, the final goods resource constraint implies that consumption is then given by:

$$C = Y\left(1 - g_{ss} - s_{ss}\right) \tag{B.10}$$

where $g_t \equiv \frac{G_t}{Y_t}$ is the share of government consumption in output. Using (B.10), the consumer's first order conditions, and the fact that $w_t = (1 - \alpha - v)Y_t/L_t$, one can express:

$$\frac{\phi l_{ss}}{\left(1 - \phi l_{ss}\right)} = \frac{(1 - \alpha - v)(1 - \tau_{lss})}{\gamma\left(1 - g_{ss} - s_{ss}\right)}$$

Finally, letting $\pi_t \equiv \frac{\Omega_t}{Y_t}$, one can rearrange terms and substitute into (B.9) to obtain:

$$MCF_{ss} = 1 + \tau_{lss}^*\left[A - (1 - \tau_{lss})\left[A + 1 + \frac{\pi_{ss}}{\left(1 - g_{ss} - s_{ss}\right)} - \frac{(1 - \alpha - v)(1 - \tau_{lss})}{\gamma\left(1 - g_{ss} - s_{ss}\right)}\right]\right]^{-1} A$$

where:

$$A \equiv (1 - \sigma)\left[1 + \frac{(1 - \alpha - v)(1 - \tau_{lss})}{\left(1 - g_{ss} - s_{ss}\right)}\right] - \sigma\frac{\pi_{ss}}{\left(1 - g_{ss} - s_{ss}\right)}$$

Note that π and g will be constant on a balanced growth path because the levels of government spending on consumption and transfers grow at the rate of technological progress, g_x.[59] Finally, one can solve for s_{ss} in the following steps. First, use the final consumption good resource constraint to express s_{ss} as a function of the capital-output ratio:

$$G_t + C_t + K_{t+1} = (1 - \delta)K_t + Y_t$$
$$g_t Y_t + (1 - g_t - s_t) Y_t + K_{t+1} = (1 - \delta)K_t + Y_t$$
$$s_{ss} = \frac{K_{ss}}{Y_{ss}}\left[(1 + g_x) - (1 - \delta)\right] \tag{B.11}$$

where the last equation follows from the fact that, on a balanced growth path, $K_{t+1} = K_t(1+g_x)$. Next, evaluate the agent's Euler equation at the utility function (41). Noting that, on a balanced growth path, $L_t = L_{t+1}$ and $C_{t+1} = C_t(1+g_x)$, one can rearrange to solve for the balanced growth path capital-output ratio:

$$\alpha\left[\frac{(1 + g_x)^\sigma}{\beta} - (1 - \delta)\right]^{-1} = \frac{K_{ss}}{Y_{ss}} \tag{B.12}$$

where I set $\tau_{k_{ss}}^* = 0$, given the form of preferences (41). Finally, combining (B.11) and (B.12) yields an expression for s_{ss} :

$$s_{ss} = a\left[\frac{(1 + g_x)^\sigma}{\beta} - (1 - \delta)\right]^{-1}\left[(1 + g_x) - (1 - \delta)\right]$$

---

[59] I assume zero population growth in the very long run.

To summarize, one can thus express the MCF along a balanced growth path as a function of the model parameters and three key fiscal variables: g_{ss}, s_{ss}, and τ^*_{lss}. However, these fiscal variables are endogenous and depend on the entire sequence of tax rates and revenues collected, implying that the MCF cannot be fully solved for in closed form without further restrictions on preferences.

C   Appendix C

To assess the quantitative effects and importance of non-renewable resource dynamics for optimal carbon taxes, I integrate the fossil fuel energy production sector of Golosov, Hassler, Krusell, and Tsyvinski (2011) into my model. Specifically, I replace the energy sector of the baseline calibration with the following three energy resources:

Name	Stock	Production	Notes
Coal	Unlimited	Labor Inputs[†]	Productivity growth as in final goods sector
Oil	400 GtC	Costless	$5 \times$ more efficient than coal per ton C
Backstop	Infinite	Costless	Available in 2120

[†]Worker productivity in coal extraction is calibrated to match a coal price of $74/$mt$ in 2009 (GHKT, 2011).

Following GHKT (2011), I implement this technological shift by changing the production function to not require energy inputs after 2120.

Table 8: Energy Sector

Note that there is no clean energy or abatement technology in this calibration until carbon energy becomes obsolete in 2120. Oil delivers more energy per ton of carbon content, and is costless to extract. Consequently, the optimal energy production trajectory begins with an oil-only regime, which lasts until economically viable petroleum reserves are exhausted.[60] A coal regime follows, until alternative energy forms become available in 2120.

As discussed above, optimal carbon taxes during the oil regime are only uniquely determined relative to an assumed initial tax, $\overline{\tau_{E0}}$. That is, oil producers' behavior depends only on the relative rates of return to extraction across time periods. As a result, the optimal emissions path during the oil regime can be implemented by many carbon tax schedules. However, for a given initial tax $\overline{\tau_{E0}}$, the carbon price path that decentralizes the optimal allocation is pinned down by the oil producers' Hotelling condition (HOT). I consider two levels of $\overline{\tau_{E0}}$, which represents

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[60]   In alternative calibrations, it may not be desirable to use up all petroleum (see Golosov, Hassler, Krusell, and Tsyvinski, 2011, for a discussion). However, within the context of my model, I find that oil is exhausted in all scenarios considered.

the carbon tax in place from 2015-2025. The first is $59.60/mtC$, which is the optimal carbon tax identified by Golosov, Hassler, Krusell, and Tsyvinski (2011) for the year 2010 within the context of their model. The second is 50% lower at $28.5/mtC$.

The second key tax parameter for a model with non-renewable resources is the feasible tax rate on Hotelling profits. Empirical estimates of effective tax rates levied on Hotelling profits from oil production vary greatly, from less than 30% in Ireland to over 90% in Iran (Johnston, 2008). I thus consider the two limiting cases that the governments taxes Hotelling profits either fully ($\tau_\pi = 100\%$) or not at all ($\tau_\pi = 0\%$).

In sum, the following five model scenarios are considered:

1. *First-best with lump-sum taxes.* There are no distortionary taxes in this scenario, as is commonly assumed in the literature.

2. *Full profit taxes available.* This scenario features optimized distortionary taxes. The government can fully tax away profits from oil extraction.

3. *No profits taxes, $\overline{\tau_{E0}} = \$56.9/mtC$.* Distortionary taxes are optimized. However, the government cannot tax oil profits directly. The initial carbon tax now affects the rents households receive (see equation (54)), and is set at $56.9/mtC$.

4. *No profits taxes, $\overline{\tau_{E0}} = \$28.5/mtC$.* Identical to (3) but with the initial carbon tax set at $28.5/mtC$.

Table (9) summarizes the results.

Scenario	Capital Tax	Labor Tax	Carbon Tax		MCF	T_t
	Avg.	Avg.	$/mtC$		Avg.	$C°$
	2025-2255	2025-2255	2065[†]	2105[†]	2025-2255	Max
First-best (Lump-Sum Taxes)	0.0%	0.0%	247	470	1.000	4.44
Full profits taxes available	2.73%	42.2%	228	430	1.058	4.48
No profits taxes, $\tau_{E,2015} = \$56.9/mtC$	3.0%	43.3%	219	420	1.060	4.85
No profits taxes, $\tau_{E,2015} = \$28.5/mtC$	3.1%	43.9%	215	415	1.062	4.98

[†]These taxes pertain to the coal regime after oil reserves have been exhausted.
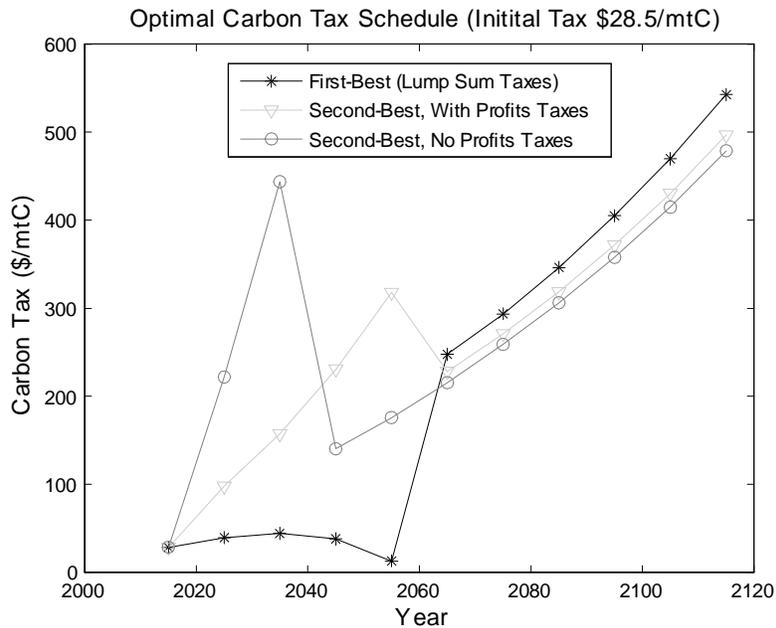
Table 9: Fossil Fuel Results

These figures are broadly in line with the benchmark calibration, and suggest three main quantitative results.
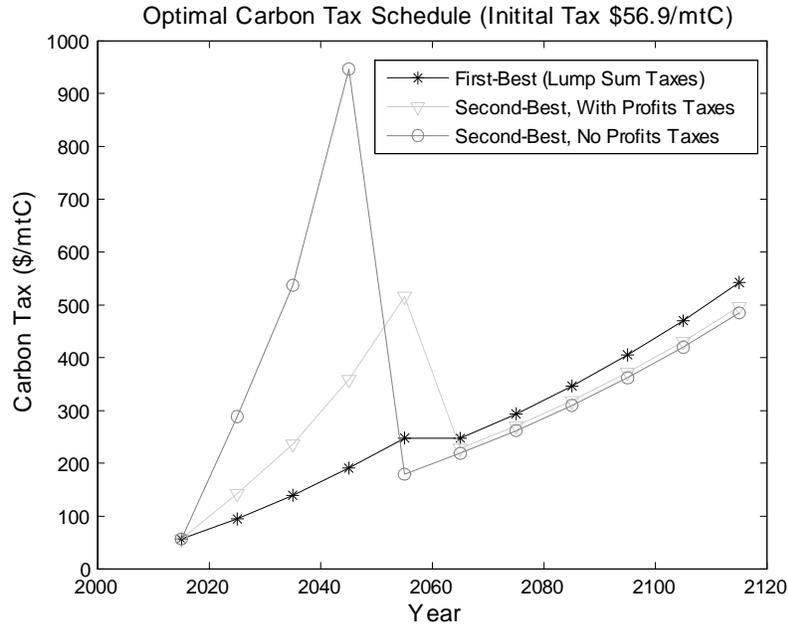
First, optimal peak temperature change is higher when there are distortionary taxes. Figure (C) displays optimal temperature change across model scenarios.
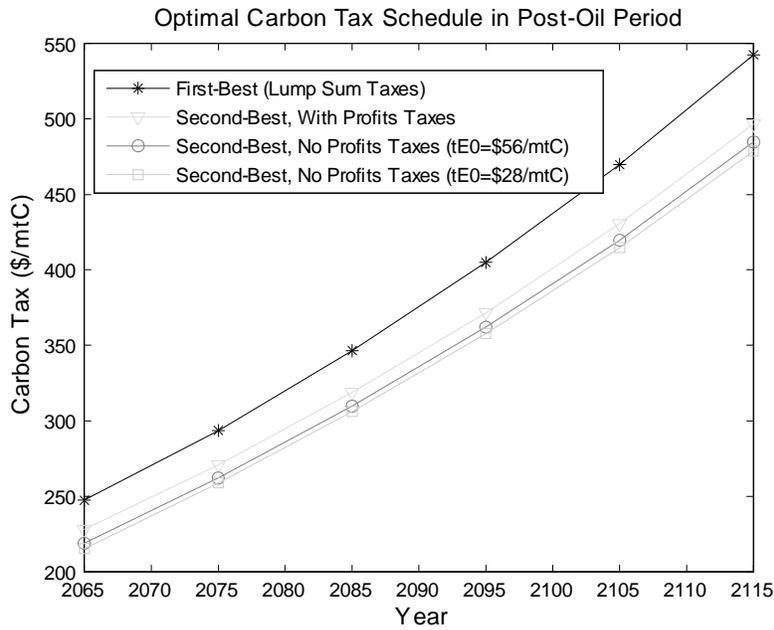
Optimal Temperature Change
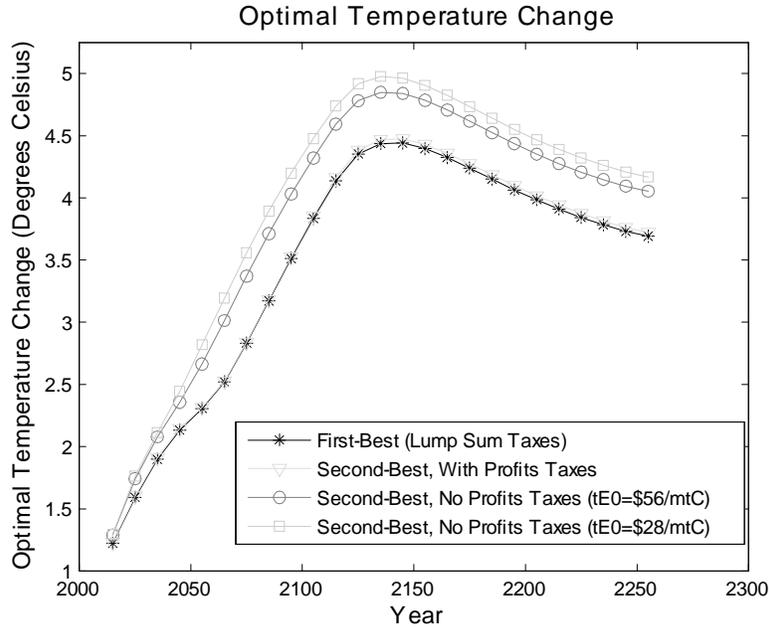
Note that the absolute level of optimal temperature change is significantly higher across all model scenarios for the GHKT (2011) energy sector calibration than in the benchmark COMET results. An important difference is that there is no alternative energy or emissions abatement technology available until 2120 in this model extension.

Second, optimal carbon taxes during the oil regime depend critically on the initial tax, and may be considerably *higher* in the setting with distortionary taxes, particularly if oil company profits cannot be taxed directly. Figures (C) and (C) display optimal carbon price paths for the two initial tax rates considered, $\tau_{E0} = \$56.9/mtC$ and $\tau_{E0} = \$28.4/mtc$.

Optimal Carbon Tax Schedule (Initital Tax $28.5/mtC)

Optimal Carbon Tax Schedule (Initital Tax $56.9/mtC)

Third, during the coal regime, optimal carbon levies are consistently lower when there are distortionary taxes. On average, they are 8% to 12% lower in the setting with fully optimized distortionary taxes. Figure (C) compares carbon prices during these years:

Optimal Carbon Tax Schedule in Post-Oil Period

In summary, the three main quantitative results from the extension to non-renewable energy resources are as follows: (1) Optimal peak temperature change (in $°C$) is between 1% and 12% higher when there are distortionary taxes; (2) Optimal carbon taxes during the oil regime are not uniquely determined but can be considerably higher when there are distortionary taxes,

particularly if profit taxes are not available; (3) Optimal carbon taxes during the coal regime are between 8% and 12% lower when there are optimized distortionary taxes.

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